Dynamic Interactions Between Health, Human Capital and Wealth

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Abstract

This paper proposes a dynamic economic model with health, human capital and wealth accumulation with elastic labor supply. The economic system consists of one industrial, one health, and one education sector. Our model is a synthesis of four main models in economic theory: Solow’s one-sector neoclassical growth mode, the Uzawa-Lucas two sector model, Arrow’s learning by doing model, and Grossman’s growth model with health. The model also includes Zhang’s idea about creative leisure or learning by consuming. Demand and supply of health service and education are determined by market mechanism. The model describes dynamic interdependence among wealth, health, human capital, economic structure, and time distribution among work, health caring, and education under perfect competition. We simulate the model and examine effects of changes in the propensity to consume health caring, the efficiency of producing health caring, the propensity to receive education, and the propensity to save.

Keywords: health; human capital; wealth accumulation; economic structure

Introduction

The purpose of this study is to study interactions between health, human capital and economic growth. In the literature of economic growth with health, human capital often refers to the human capital accumulated through formal education. We refer to Grossman (1972) for distinguishing health and human capital. Health is generally considered a component of human capital in the literature of health economics. Worker’s productivity may be improved by physical capacities, such as strength and endurance, and mental capacities, such as cognitive functioning. Nevertheless, as pointed out by Bloom, et al. (2004:1), “most cross-country empirical studies identify human capital narrowly with education. This practice ignores strong reasons for considering health to be a crucial aspect of human capital, and therefore a critical ingredient of economic growth.” Workers with better health tend to be physically and mentally more energetic and robust. Because of their higher productivity, they earn more. The positive effect of health is identified in many empirical studies (e.g., Parkin et al., 1987; Posnett and Hitiris, 1992; Strauss and Thomas, 1998; Rivera and Currais, 1999; Schultz, 1999; and Bloom, et al. (2004). Some empirical studies demonstrate positive effects of health on economic growth (Bloom and Canning, 2003). Since Newhouse (1977) published the seminal paper about relationships between health care
expenditures and income and the magnitude of income elasticity of expenditures, many empirical studies have been conducted in measuring the income elasticities in different economies (e.g., Yavuz et al., 2011). The research results are not converged. For instance, Baltagi and Moscone (2010) find the health expenditure as a necessity good for 20 OECD countries, while Murray et al. (1994) find it a luxury good for 138 countries. In his seminal contribution to economic endogenous health, Grossman (1972) explains the importance of introducing health into economics because health affects well-being and economic growth in different ways. A good health contributes positively to labor productivity. The provision of health also requires resources. There are direct trade-offs between health and other economic activities.

It is generally agreed that human capital is a key element for economic growth (e.g., Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; and Castelló-Climent and Hidalgo-Cabrillana, 2012). In most of the literature of economic growth human capital is accumulated mainly through formal education. As the provision and consumption of education require resources, it is important to explain individual returns to education. There are many empirical studies on the return to education since Mincer (1974) published the seminal work in 1974. For instance, Fleisher et al. (2011) shows that an additional year of schooling raises marginal product by 30.1 percent, and the CEO’s education increases TFP for foreign-invested firms, using the firm-level data from China. As shown by Fleisher and Chen (1997), China’s high and rising regional income inequality is highly due to regional inequality of investment in higher education. Li et al. (2012) find that in China high school education has low returns in terms of earnings and may mainly serve as a mechanism to select students for higher education; both vocational school education and college education have a large return which is comparable to that found in the U.S. Theoretically, the work by Lucas (1988) has caused a great interest in relation between human capital and economic growth. The first formal dynamic growth model with education was proposed by Uzawa (1965). Nevertheless, in the Uzawa-Lucas model and many of their extensions and generalizations, it is implicitly assumes that all skills and human capital is formed due to formal schooling. Obviously much of the so-called human capital may be accumulated through parents’ influences and many other social and economic activities, not to say learning by producing (and professional training). Another issue is described by Chen and Chevalier (2008), “Making and exploiting an investment in human capital requires individuals to sacrifice not only consumption, but also leisure. When estimating the returns to education, existing studies typically weigh the monetary costs of schooling (tuition and forgone wages) against increased wages, neglecting the associated labor/leisure tradeoff.” As emphasized by Grossman (1972) long time ago, health caring requires time and resource. Our study is to model trade-offs between health and education.

Both health and human capital are important for understanding economic development. As van Zon and Muysken (2001: 170) emphasize, “people can provide effective human
capital services only if they are alive and healthy. Therefore, the general acceptance of human capital formation as a source of growth also warrants a closer look at how changes in the health-care of the population may influence growth and hence total welfare.” An increase in education tends to have positive effects on health (Pritchett and Summers, 1996), while an improvement in health also affects education (Pollitt, 1997). To study trade-offs among wealth accumulation, health care and education, we extend the Uzawa-Lucas two-sector growth model. We take account of health service and health caring. We build a growth model with physical capital, human capital and health. The model is based on some key models in economic theory. The economy consists of industrial, education and health sectors. As far as the industrial sector and growth mechanism with capital accumulation are concerned, we follow the Solow one-sector growth model. In modeling the relations between the industrial sector and education sector, we follow the Uzawa-Lucas two-sector growth model. We use an alternative approach by Zhang (2005) to describe behavior of the household. The human capital accumulation a synthesis of three ideas in economics—Uzawa’s learning through education, Arrow’s learning by doing model, Zhang’s learning through consumption/creative leisure (Zhang, 2007). As far as modeling the dynamics of health stock is concerned, this paper is influenced by Grossman’s approach (Grossman, 1972). We integrate these ideas and main mechanisms of economic growth in a comprehensive framework. The remainder of the paper is organized as follows. Section 2 defines the economic model with endogenous wealth accumulation, human capital accumulation and health change. Section 3 shows that the motion of the economic system is described by three differential equations and simulates the model. Section 4 carries out comparative dynamics analysis. Section 5 concludes the study.

The basic model

The economy has three sectors - industrial, health and education sectors. Most aspects of the industrial sector are similar to the standard one-sector growth model in the neoclassical growth theory (Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995; and Zhang, 2005). It is assumed that there is only one (durable) good in the economy under consideration. Households own assets of the economy and distribute their incomes to consume and save. The industrial sector uses physical and qualified labor inputs to produce goods and services. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. We assume a homogenous and fixed population \( N_0 \). The labor force is distributed among production, teaching and health caring. We select commodity to serve as numeraire, with all the other prices being measured relative to its price. We assume that the wage rate is equal among all professions. The total capital stock,
$K(t)$ is allocated to the three sectors. We use subscript index, $i$, $h$ and $e$, to stand for, respectively, industry, health and education. We use $N_j(t)$ and $K_j(t)$, to stand for the qualified labor force and capital stocks employed by sector $j$, $j = i, h, e$. We use $T(t)$, $T_h(t)$ and $T_e(t)$ to respectively stand for the work time, the time for health care and education time. As full employment of labor and capital is assumed, we have

\begin{equation}
K_i(t) + K_h(t) + K_e(t) = K(t), \quad N_i(t) + N_h(t) + N_e(t) = N(t),
\end{equation}

where $N(t)$ is the total qualified labor force. We rewrite (1) as follows

\begin{equation}
n_i(t)k_i(t) + n_h(t)k_h(t) + n_e(t)k_e(t) = k(t), \quad n_i(t) + n_h(t) + n_e(t) = 1,
\end{equation}

in which

\begin{equation}
k_j(t) = \frac{K_j(t)}{N_j(t)}, \quad n_j(t) = \frac{N_j(t)}{N(t)}, \quad k(t) = \frac{K(t)}{N(t)}, \quad j = i, h, e.
\end{equation}

We use $H(t)$ and $L(t)$ to stand for the level of human capital and health stock of the population. We specify the total qualified labor force as follows

\begin{equation}
N(t) = T(t)H^m(t)L^{m_h}(t)N_0,
\end{equation}

in which the parameters, $m$ and $m_h$ measure the efficiencies that the worker applies human capital and health. According to Weil (2007) and Tobing (2011), the total labor force is given by $N(t) = T(t)H(t)l(t)N_0$, which is a special case of (3) with $m = 1$ and $m_h = 1$.

The three sectors

We use the conventional production functions to describe the relationships between inputs and output. The production functions of the three sectors are

\begin{equation}
F_j(t) = A_jK_j^{\alpha_j}(t)N_j^{\beta_j}(t), \quad A_j, \alpha_j, \beta_j > 0, \quad \alpha_j + \beta_j = 1, \quad j = i, h, e,
\end{equation}

where $A_j$, $\alpha_j$, and $\beta_j$ are parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest $r(t)$ and
wage rate \( w(t) \) are determined by markets. We use \( p_j(t) \) to stand for the price of sector \( j \), with \( p_j(t) = 1 \). For any individual firm \( r(t) \), \( w(t) \) and \( p_j(t) \) are given at each point of time. Sector \( j \) chooses \( K_j(t) \) and \( N_j(t) \) to maximize its profit. The marginal conditions are

\[
 r(t) + \delta_k = \alpha_j A_j p_j(t) k_j^{-\beta_j}(t), \quad w(t) = \beta_j A_j p_j(t) k_j^{\alpha_j}(t),
\]

where \( \delta_k \) is the fixed depreciation rate of physical capital.

**The quality of health caring**

In deciding what enters the utility function, we follow the approach of treating health service as a commodity accepted by Grossman (1972), “Consumers produce commodities with inputs of market goods and their own time. For example, they use traveling time and transportation services to produce visits; part of their Sundays and church services to produce ‘peace of mind’; and their own time, books, and teachers’ services to produce additions to knowledge.” We consider that the output of health caring, \( c_h(t) \), is dependent on the level of health service (which consumer purchases in market) and the consumer’s time spent on health caring. To maintain health, the consumer may do physical exercises such as running and walking, go to hospitals for health checking, taking medicines for curing illness, and so on. We assume that the health caring quality is dependent on the consumption level of health service, \( c_h(t) \), and the time spent on health caring, \( T_h(t) \), in the following way

\[
 c_h(t) = \overline{A} c_h^\overline{\alpha}(t) T_h^\overline{\beta}(t), \quad \overline{A}, \overline{\alpha}, \overline{\beta} > 0,
\]

where \( \overline{A} \), \( \overline{\alpha} \), and \( \overline{\beta} \) are parameters.

**Consumer behaviors**

Consumers choose consumption levels of commodity, health and education, and make decision on how much to save. We denote per capita wealth by \( \overline{k}(t) \), where \( \overline{k}(t) \equiv K(t)/N_0 \). The per capita current income from the interest payment \( r(t)\overline{k}(t) \) and the wage income \( T(t)H^m(t)l^m(t)w(t) \) is

\[
y(t) = r(t)\overline{k}(t) + T(t)H^m(t)l^m(t)w(t).
\]
We call $y(t)$ the current income in the sense that it comes from the consumer’s wage income and current earnings from ownership of wealth. The amount of purchasing resource that the consumer uses for consuming, saving, and education is not necessarily equal to the temporary income because the consumer can sell wealth to pay, for instance, the current consumption if the temporary income is not sufficient for buying food and touring the country. Here, we neglect the possibility of borrowing from other places. The total value of wealth that the consumer can sell to purchase goods and to save is equal to $p_i(t)k(t)$, with $p_i(t) = 1$. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is given by

$$\tilde{y}(t) = y(t) + k(t).$$  \hspace{1cm} (8)

The disposable income is used for saving, consumption, health caring, and education. The consumer distributes the total available budget among consumption of goods, $c(t)$, saving, $s(t)$, health service, $p_h(t)c_h(t)$, and education, $p_e(t)T_e(t)$. The budget constraint is

$$c(t) + s(t) + p_h(t)c_h(t) + p_e(t)T_e(t) = \tilde{y}(t) = (1 + r(t))k(t) + T(t)H_m(t)l_{m_h}(t)w(t).$$  \hspace{1cm} (9)

The consumer is faced with the following time constraint

$$T(t) + T_h(t) + T_e(t) = T_0,$$  \hspace{1cm} (10)

where $T_0$ is the total available time. Substituting (10) into (9) yields

$$c(t) + s(t) + p_h(t)c_h(t) + \bar{p}_h(t)T_h(t) + \bar{p}_e(t)T_e(t) = \bar{y}(t),$$  \hspace{1cm} (11)

in which

$$\bar{p}_h(t) \equiv H^m(t)l_{m_h}(t)w(t), \quad \bar{p}_e(t) \equiv H^m(t)l_{m_e}(t)w(t) + p_e(t),$$

$$\bar{y}(t) \equiv (1 + r(t))k(t) + T_0 H^m(t)l_{m_h}(t)w(t).$$

The variable $\bar{p}_h(t)$ is the opportunity cost of health caring, $\bar{p}_e(t)$ is the opportunity cost of education, and $\bar{y}(t)$ is the potential income that the household can have by spending all the available time on working.

In our model, at each point of time, consumers have four variables, the level of consumption, the level of saving, the level of health caring, and the education time, to decide. The utility is a function of $c(t)$, $s(t)$, $\tilde{c}(t)$, and $T_e(t)$. We specify the utility function as follows

$$U(t) = c^{\xi_0}(t)s^{\lambda_0}(t)\tilde{c}^{\psi_0}(t)T_e^{\eta_0}(t), \quad \xi_0, \lambda_0, \psi_0, \eta_0 > 0,$$  \hspace{1cm} (12)
where $\xi_0$ is called the propensity to consume, $\lambda_0$ the propensity to own wealth, $\psi_0$ the propensity to use health care service, and $\eta_0$ the propensity to receive education. A detailed explanation of the utility function and its applications to different dynamic problems are provided in Zhang (2005, 2007). It should be noted that we assume that health caring $c(t)$, rather than health service $c_t(t)$, enters the utility function. As explained by Grossman (1972), “what consumers demand when they purchase medical services are not these services per se but, rather, ‘good health.’ Given that the basic demand is for good health, it seems logical to study the demand for medical care by first constructing a model of the demand for health itself.” It should be noted that we assume that the education time directly enters the utility function. In traditional economic growth theory with endogenous human capital, education is mainly modeled by assuming that it positively affects earnings through enhanced productivity. Nevertheless, it is argued that one chooses education not only for higher wages, but also for social status, for social network buildings, signaling, or other purposes. The signaling view of education was initially formally presented by Spence (1973), Arrow (1973), and Stiglitz (1975). From this view we know that it may not proper to use direct productivity gains to explain the choice of quantity and quantity of education. As explained by Lee (2007) by the signaling view of education may partly explain why American students study more in college than in high school while the opposite is true for East Asian students. Hussey (2012) empirically shows that signaling plays a large role in producing post-graduation earnings, using U.S. data.

Maximizing $U(t)$ subject to (6) and (11) yields

$$c(t) = \xi \bar{y}(t), \quad s(t) = \lambda \bar{y}(t), \quad p_h(t)c_h(t) = \bar{\psi} \bar{y}(t), \quad \bar{p}_h(t)T_h(t) = \bar{\psi} \bar{y}(t), \quad \bar{p}_e(t)T_e(t) = \eta \bar{y}(t), \quad \text{(13)}$$

where

$$\xi = \xi_0 \rho, \quad \lambda = \lambda_0 \rho, \quad \bar{\psi} = \bar{\alpha} \psi_0 \rho, \quad \bar{\psi} = \bar{\beta} \psi_0 \rho, \quad \eta = \eta_0 \rho, \quad \rho = \frac{1}{\xi_0 + \lambda_0 + \bar{\alpha} \psi_0 + \bar{\beta} \psi_0 + \eta_0}.$$

Wealth accumulation

According to the definition of $s(t)$, the change in the household’s wealth is given by

$$\dot{k}(t) = s(t) - \bar{k}(t) = \lambda \bar{y}(t) - \bar{k}(t). \quad \text{(14)}$$

The equation says that the change in wealth is the saving minus dissaving.

Demand and supply of the three sectors

For the education sector, the demand and supply balances at any point of time

$$T_e N_0 = F_e(t). \quad \text{(15)}$$
For the health sector, the demand and supply balances at any point of time
\[ c_h(t)N_0 = F_h(t). \] (16)

As output of the industrial sector is equal to the sum of the level of consumption, the depreciation of capital stock and the net savings, we have
\[ C(t) + S(t) - K(t) + \delta_K K(t) = F_i(t), \] (17)

where \( C(t) \) is the total consumption, \( S(t) - K(t) + \delta_K K(t) \) is the sum of saving and depreciation. We have
\[ C(t) = c(t)N_0, \quad S(t) = s(t)N_0. \]

**Human capital accumulation**

Human capital accumulation is a continuous process over individual lifetime. We assume that there are three sources of improving human capital: learning through education, “learning by producing”, and “learning by leisure”. Arrow (1962) first introduced learning by doing into growth theory; Uzawa (1965) took account of trade-offs between investment in education and capital accumulation, and Zhang (2007) introduced impact of consumption on human capital accumulation (via the so-called creative leisure) into growth theory.

We use \( \bar{f}_e \) and \( \bar{f}_i \) to stand for the output levels of the education and industrial sectors per capita, that is
\[ \bar{f}_e(t) \equiv \frac{F_e(t)}{N_0}, \quad \bar{f}_i(t) \equiv \frac{F_i(t)}{N_0}. \]

The human capital dynamics is specified by
\[ \dot{H}(t) = \frac{\nu_e \bar{f}_e^{a_e}(t)}{H^{\pi_e}(t)} + \frac{\nu_i \bar{f}_i^{a_i}(t)}{H^{\pi_i}(t)} + \frac{\nu_h h(t) c_i^{a_h}(t)}{H^{\pi_h}(t)} - \delta_h H(t), \] (18)

where \( \delta_h (> 0) \) is the depreciation rate of human capital, \( \nu_e, \nu_i, \nu_h, a_e, a_i, a_h \) and \( b_h \) are non-negative parameters. The signs of the returns-to-scale parameters \( \pi_e, \pi_i, \) and \( \pi_h \) are not specified as they can be either negative or positive.

The above equation is a synthesis and generalization of Arrow’s, Uzawa’s, and Zhang’s ideas about human capital accumulation. The term, \( \nu_e \bar{f}_e^{a_e} / H^{\pi_e} \), describes the contribution to human capital improvement through education. Human capital rises with an increase in the level of education service, \( f_e \), each student receives. The term
\( H^{\pi_e} \) indicates that as the level of human capital of the population increases, it may be more difficult (in the case of \( \pi_e > 0 \)) or easier (in the case of \( \pi_e < 0 \)), for instance, due to learning externalities as in Choi (2011) to accumulate more human capital via formal education. Rauch (1993) and Liu (2007) study human capital externalities. Cohn and Cooper (2004) examine economies of scale and scope in education. According to Biagetti and Sergio (2009), empirical literature does not seem to take sufficiently into account the contribution of learning through working as an additional source of human capital and growth. We take account of learning by doing effects in human capital accumulation by the term \( \nu_i \tilde{f}_i^{a_i} / H^{\pi_i} \). This term implies that contribution of the industrial sector to human capital improvement is positively related to the per capita output. The term \( H^{\pi_i} \) takes account of returns to scale effects in human capital accumulation. We take account of learning by consuming by the term \( \nu_h l^{b_h} c^{a_h} / H^{\pi_h} \). This term can be interpreted similarly as the term for learning by producing.

In the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (see Barro and Sala-i-Martin, 1995)

\[
\dot{H}(t) = H(t)G(T_e(t)),
\]

where the function \( G \) is increasing as the effort rises with \( G(0) = 0 \). This formation is due to Lucas (1988). Uzawa’s model is a special case of the Lucas model with \( \gamma = 0 \), \( U(c) = c \), and the assumption that the right-hand side of the above equation is linear in the effort. Solow (2000) uses

\[
\dot{H}(t) = H(t)\kappa T_e(t).
\]

The new formation implies that if no effort is devoted to human capital accumulation, then \( \dot{H}(0) = 0 \) (human capital does not vary as time passes; this results from depreciation of human capital being ignored); if all effort is devoted to human capital accumulation, then \( g_H(t) = \kappa \) (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). As explained by Solow (2000), the formulation is very far from a plausible relationship. If we consider the above equation as a production for new human capital, the production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to \( H(t) \) itself. It can be seen that our approach in (18) is more general than the traditional formation with regard to education.
Health dynamics

We consider that change in health stock is dependent on nutrition and health caring. We propose the health dynamics as follows

$$\delta_h(t) = \frac{\bar{U}_h c^{\pi_h}(t)}{\bar{v}_h}(t) + \frac{\bar{U}_c c^{\pi_c}(t)}{\bar{v}_c}(t) - \delta_h(t),$$  \hspace{1cm} (19)$$

where $\delta_h (> 0)$ is the depreciation rate of health stock, $\bar{U}_h$, $\bar{U}_c$, $\bar{a}_h$, and $\bar{a}_c$ are non-negative parameters. The signs of the returns-to-scale parameters $\pi_h$ and $\pi_c$ are not specified as they can be either negative or positive. The term $\bar{U}_h c^{\pi_h} / \bar{v}_h$ means that the improvement health is positively related to the investment in health caring and there are returns to scale effect in health improvement due to investment in health caring. The term $\bar{U}_c c^{\pi_c} / \bar{v}_c$ means that the improvement health is positively related to the consumption level and there are returns to scale effect in health improvement due to nutrition intake. The term reflects that housing, diet, travels, cigarette smoking, and alcohol consumption may affect health. As pointed out by Grossman (1972), the rate of depreciation might be a negative function of the health stock. As shown later, any functional form will not affect our simulation.

It should be noted that change in health is modeled in different ways. For instance, according to Tobing (2011), health status $\hat{I}$ is given by

$$\hat{I}(t) = \nu\left(\frac{g(t)}{y(t)}\right)^{\xi}, \hspace{1cm} \nu, \xi > 0, \hspace{0.5cm} \xi < 1,$$

where $g(t)$ and $y(t)$ are respectively per capita public expenditure on health and current income, $\nu$ and $\xi$ respectively measures the productivity of medical care technology and the elasticity of health investment. According to Johansson and Löfgren (1995), health stock evolves by the following equation

$$\delta_h(t) = g(z(t), k_h(t)) - \delta_h(t),$$

where $z(t)$ is the level of pollutants and $k_h(t)$ is the capital input for health caring. Our approach is similar to Johansson and Löfgren, even though we don’t explicitly take account of pollution. Grossman (1972) uses the same dynamic equation with $g(t)$ dependent on the time input to health caring, the level of human capital and the expenditure on medical care. In Grossman’s approach, medical care is treat as the most important market good in the investment function. It should be also noted that
in our approach to human capital accumulation and health dynamics, we don’t specify increasing or decreasing returns to scale like, for instance, in von Zon and Muysken (2011), who assume that the generation of health services is characterized by decreasing returns, whereas human capital accumulation exhibits increasing returns. Each source of learning or health caring may exhibit either increasing or decreasing returns to scales.

We thus built the dynamic model. We now examine dynamics of the model.

The dynamics and its properties

We study dynamics of the model. The following lemma shows the procedure of how to determine the motion of all the variables in the economic system over time. Before describing the procedure we introduce a new variable $z(t) = \left( r(t) + \delta_h \right) / w(t)$.

Lemma 1

The dynamics of the economic system is governed by the following three differential equations

$$
\begin{align*}
\dot{H}(t) &= \Omega_z(z(t), H(t), l(t)), \\
\dot{L}(t) &= \Omega_H(z(t), H(t), l(t)), \\
\dot{L}(t) &= \Omega_l(z(t), H(t), l(t)),
\end{align*}
$$

with $z(t)$, $H(t)$ and $l(t)$ as the variables. The three functions, $\Omega_z$, $\Omega_H$, and $\Omega_l$ are functions of $z(t)$, $H(t)$ and $l(t)$ defined in the appendix. Moreover, all the other variables are determined as functions of $z(t)$, $H(t)$ and $l(t)$ at any point of time by the following procedure: $k_j$ by (A1) $\rightarrow r$ and $w$ by (A2) $\rightarrow p_e$ and $p_h$ by (A3) $\rightarrow T$ by (A13) $\rightarrow \bar{y}$ by (A14) $\rightarrow \bar{k} = kTH^{m_1}m_h \rightarrow c$, $s$, $c_h$, $T_h$, and $T_e$ by (13) $\rightarrow n_i$ and $n_e$ by (A10) $\rightarrow n_h$ by (A8) $\rightarrow \bar{c}$ by (8) $\rightarrow K = \bar{k}N_0 \rightarrow N$ by (3) $\rightarrow N_j = n_jN \rightarrow K_j = k_jN_j \rightarrow F_j$ by (4).

The three differential equations in (20) contain three variables, $z(t)$, $H(t)$ and $l(t)$. Although we can analyze its dynamic properties as we have explicitly expressed the dynamics, we omit analyzing the model as the expressions are too complicated. We simulate the model to show the motion of the system. In the remainder of this study, we specify $T_0 = 1$ and the population $N_0 = 10$. The requirement $T_0 = 1$ will not affect our analysis. We specify the production function parameters as follows

$$
\begin{align*}
\alpha_e &= 0.3, \quad \alpha_s = 0.34, \quad \alpha_h = 0.4, \quad A_e = 1, \quad A_s = 0.9, \quad A_h = 1.1, \quad \delta_e = 0.05, \quad \delta_s = 0.04, \quad \bar{\delta}_h = 0.06.
\end{align*}
$$
We specify the values of the parameters, \( \alpha_j \), in the Cobb-Douglas productions approximately equal to 0.3. These parameter values are often used in empirical studies (for instance, Miles and Scott, 2005; Abel et al. 2007). The total productivities of the three sectors are specified \( A_i = 1, A_e = 0.9, \) and \( A_h = 1.1 \). The depreciation rate of physical capital is often fixed around 0.05 in economic studies. According to Stokey and Rebelo (1995), it is reasonable to consider the depreciation rate of human capital a range between 0.03 and 0.08 for the US economy. The parameters for the household preference and efficiencies of human capital and health are specified as follows

\[
\bar{\alpha} = 0.5, \quad \bar{\beta} = 0.3, \quad \lambda_o = 0.8, \quad \xi_o = 0.12, \quad \psi_o = 0.04, \quad \eta_o = 0.012, \quad m = 0.6, \quad m_h = 0.7.
\]

The condition \( \bar{\alpha} = 0.5 \) and \( \bar{\beta} = 0.3 \) implies decreasing returns in health caring. Decreasing returns in health services are used in studies by, for instance, Forster (1989), Ehrlich and Chuma (1990), Johansson and Löfgren (1995), and van Zon and Muysken (2001). The propensity to save is 0.8 and the propensities to consume goods, to maintain health, and to receive education are respectively 0.12, 0.04, and 0.012. The implications of the specified parameter values are that the household would like to spend on consuming goods three times as much as on purchasing health service, and would like to spend on purchasing health service approximately three times as much as on receiving education. We specify the parameters in the dynamic equations for human capital and health stock as

\[
\begin{align*}
v_c &= 0.8, \quad v_i = 2.5, \quad v_h = 0.7, \quad a_e = 0.3, \quad b_e = 0.5, \quad a_i = 0.4, \quad a_h = 0.1, \quad b_h = 0.3, \quad \pi_e = 0.2, \\
\pi_i &= 0.7, \quad \pi_h = 0.5, \quad \bar{v}_h = 0.5, \quad \bar{v}_c = 0.3, \quad \bar{a}_h = 0.3, \quad \bar{a}_c = 0.4, \quad \bar{\pi}_c = 0.6, \quad \bar{\pi}_h = 0.2.
\end{align*}
\]

The conditions \( \pi_e = 0.2, \quad \pi_i = 0.7, \) and \( \pi_h = 0.5 \) imply that the learning by education, learning by producing, and learning by consuming exhibits decreasing effects in human capital accumulation. Similarly, the conditions \( \bar{\pi}_c = 0.6 \) and \( \bar{\pi}_h = 0.5 \) imply that the health caring and nutrition exhibit decreasing effects in health improving. To simulate the model, we specify the initial conditions

\[
z(0) = 0.07, \quad H(0) = 33, \quad \ell(0) = 6.
\]

The simulation result is plotted in Figure 1. The rate of interest and the working time fall over time. The rest variables rise over time before they approach their equilibrium values.
The simulation demonstrates that the dynamic system has a unique equilibrium point. We list the equilibrium values of the variables as follows:

\[ H = 36.16, \quad \ell = 7.30, \quad N = 298.54, \quad K = 244.99, \quad r = 0.02, \quad p_h = 0.72, \quad p_e = 1.02, \quad w = 1.31, \]
\[ N_l = 262.05, \quad N_c = 0.41, \quad N_h = 28.01, \quad n_l = 0.902, \quad n_c = 0.001, \quad n_h = 0.097, \quad K_l = 2090.63, \]
\[ K_c = 3.92, \quad K_h = 348.44, \quad F_l = 488.60, \quad F_c = 0.79, \quad F_h = 84.58, \quad k_l = 7.98, \quad k_h = 12.4, \]
\[ k_c = 9.59, \quad \bar{k} = 244.3, \quad c = 36.7, \quad c_h = 8.46, \quad \bar{c} = 1.1, \quad T = 0.84, \quad T_c = 0.08, \quad T_h = 0.08. \]

Most of the labor force is distributed to the industrial sector. The household spends almost the same hours on education and health caring. Basing on the lemma and using the equilibrium values, we calculate the three eigenvalues as follows: \(-0.20, -0.10,\) and \(-0.05\). As the three eigenvalues are negative, the unique equilibrium is locally stable, as demonstrated in Figure 1. The system always approaches its equilibrium if it is not far from the equilibrium.

**Comparative dynamic analysis in some parameters by simulation**

We demonstrated how to follow the motion of the system and showed that the dynamic system has a unique stable equilibrium. We examine how changes in some parameters affect the dynamic path of the economic system.

**The propensity to consume health caring**

First, we examine the case that the propensity to consume health caring is increased as follows: \(\psi_h : 0.04 \rightarrow 0.05\). The simulation results are plotted in Figure 2. In the plots, a variable \(\Delta x(t)\) stands for the change rate of the variable, \(x(t)\), in percentage due to
changes in the parameter value. We will use the symbol \( \Delta \) with the same meaning when we analyze other parameters. In order to examine how each variable is affected over time, we should follow the motion of the entire system as each variable is related to the others in the dynamic system. When \( \psi_0 \) is increased, the consumer consumes more the health care. As the demand for health caring is increased, the price of health is increased. The time for health caring and the consumption level of health service are increased. The total supply and the two inputs are of the health sector are increased in association with the rise in the demand. The health stock is increased. Fewer hours are spent on work and education. As the resources are shifted to the health sector, the labor and capital inputs and output levels of the education and industrial sectors are reduced. The total qualified labor force and national wealth are reduced. In association of falling in the national wealth, the rate of interest is increased. As more time and resources are shifted to the health caring, the price of education is increased and the human capital is reduced. In association with falling in human capital, the wage rate of qualified labor is reduced. The capital intensities of the three sectors are reduced.

![Figure 2 A rise in the propensity to consume health caring](image)

**The efficiency of producing health caring**

As people have more knowledge about health and put more attention to health caring, they tend to more effectively improve and maintain their health by properly combining health service and physical exercises. The increased life expectancy in different countries in modern times is contributed not only to new medicines and hospital conditions, but also to increased awareness of and knowledge about health caring. We measure this kind of (exogenous) changes by considering changes in the total productivity of health caring, \( \bar{A} \). We consider an improvement in the total
productivity of health caring: $\bar{A} : 0.8 \Rightarrow 1.2$. As the total productivity is increased, the level of health caring is improved, which also results in the rise in the health stock. The increased health stock raises the productivity, resulting in the increase of the total labor force. As the disposable income is increased, the household spends more on consumption and has more wealth. The national wealth is increased as the individual wealth is increased. As initially the total labor force rises faster than the national wealth, the capital intensities fall. In association of falling in the capital intensities, the wage rate is reduced and rate of interest is increased. When the system approaches its equilibrium point, the total labor force and national wealth are changed in the same levels. Accordingly, in the long term the capital intensities, wage rate and rate of interest are not affected. From (A3), we know that the change directions in $p_j$ ($j = i, h$) is the same as in $k_i^{\beta_i - \beta_i}$. As we specified $\beta_j - \beta_i < 0$ in our simulation, the change direction in the prices is the opposite to that of the wage rate. The prices of education and health service are increased initially and are not affected in the long term. As the prices initially rise, the time spent on education and health caring fall. In the long term the time on health caring is not affected but the time on education is increased slightly. We also simulate the case of $\bar{v}_h : 0.5 \Rightarrow 0.55$. The effects are similar to those in Figure 3.

It is should be remarked that from Figures 2 and 3, we also demonstrate what van Zon and Muysken (2001) found: “a slow down in growth may be explained by a preference for health that is positively influenced by a growing income per head, or by an ageing population. Growth may virtually disappear for countries with high rates of decay of health, low productivity of the health-sector, or high rates of discount.”
The propensity to receive education

We now allow the propensity to receive education to be increased as: \( \eta_0 : 0.012 \Rightarrow 0.016 \). The simulation results are plotted in Figure 4. As people are more interested in receiving education, they increase education time. As they spend more time and money on education, the education fee is slightly increased and the education sector employs more capital and people and the education sector’s output is increased. As people spend more time on formal education, their human capital is increased. Although human capital is increased, the total labor force is reduced. This occurs as the other two components, the work time and health stock, which determine the labor force are reduced. The output level and the two inputs of the education sector are increased, while the output levels and the two inputs of the industrial sector and the health sector are reduced. The wage rate and the prices of health service and education are affected but only slightly. The rate of interest falls initially but rises in the long term. The wealth and consumption levels of the industrial good and health service are all reduced.

It should be remarked that in our case an increased preference for education does not encourage economic growth. This is due to the implications of the specified parameter values that education does not exhibit positive returns to scale. According to Arrow (1973), a stronger interest in education may not lead to human capital and economic growth. The conclusion results from the assumption that students choose education also for the purpose of signaling. In the literature of education and economics, the signaling view of education was initially formally presented by Spence (1973), Arrow (1973), and Stiglitz (1975). This implies that direct productivity gains are not necessary to explain the choice of quantity and quantity of education. Our result shows that even if we consider that people increase their propensity to receive education and they increase formal education for learning, the long-run consumption and wealth levels per person are reduced. This occurs because education uses more resources but does not improve productivity sufficiently.

![Figure 4 A rise in the propensity to receive education](image)

Figure 4 A rise in the propensity to receive education
The propensity to save

We now examine the impact of a rise of the propensity to save $\lambda_c$ from 0.8 to 0.85. The results are plotted in Figure 5. As the household puts more for saving from the disposable income, the per capita is increased. A rise in the propensity to save also implies reduction in the relative values of the other propensities. The per capita consumption levels of health service, industrial good and education time are all reduced in the early period of the simulation period. As the economy accumulates more capital, the industrial sector increases the output level and uses more two inputs. As the capital and labor force are shifted to the industrial sector, the other two sectors’ outputs and inputs are all reduced in the early period. In association with increasing in national wealth, the rate of interest falls. The prices of education and health service fall and the wage rate rises. As human capital is improved and disposable income is increased, the consumption levels of health service and industrial good are increased in the long term. The health stock falls initially and rises in the long term. As the health stock, work time and human capital are increased, the total labor force is increased. It should be note in a study on the endogenous relationship between health care, life expectancy and output in a neoclassical growth model different from this study, Leung and Wang (2010) show that in long-run equilibrium savings and health care are both increased with positive economic growth, even though health care directly diverts resources away from goods production. It should be noted that in the short-run relationship between health care and savings is different from the long-run one. Our model shows the entire transition process in steady of being only concerned with the long-run equilibrium. As far as long-run relationship between saving and health care is concerned, our model predicts the similar conclusions to what which are consistent with some observed stylized development patterns across countries as explained by Leung and Wang’s paper with an alternative framework.

Figure 5 A rise in the propensity to save
Concluding remarks

This paper constructed a dynamic economic model with endogenous wealth, health and human capital accumulation. The economic system consists of three sectors. There are three ways of accumulating human capital: learning by producing, learning by education, and learning by consuming. The health change is affected by health caring and consumption. The labor and capital resources are allocated among the three sectors under perfectly competitive conditions. The model builds a dynamic interdependence among wealth, health and human capital with time distribution among work, accumulation, and division of labor under perfect competition. We simulated the model to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in the propensity to consume health caring, the efficiency of producing health caring, the propensity to receive education, and the propensity to save. We also got some insights into the complexity of trade-offs among health caring, education and growth. Our results demonstrate that under certain conditions, for instance, “a slow down in growth may be explained by a preference for health that is positively influenced by a growing income per head, .... Growth may virtually disappear for countries with high rates of decay of health, low productivity of the health-sector, or high rates of discount.” (van Zon and Muysken, 2001). The model may be generalized in some directions. For instance, it is reasonable to introduce government intervention in education and health into the model. Another interesting extension is to examine how health, human capital and education interact with population dynamics with endogenous birth and death rates.

Appendix: Proving Lemma 1

We now show that the dynamics can be expressed by three differential equations. From (4) and (5), we obtain

\[ z = \frac{r + \delta_k}{w} = \frac{\alpha_j}{k_j}, \]

where \( \alpha_j \equiv \alpha_j / \beta_j \). We omit time index in appendix. From (A1), we can express \( k_j \) as functions of \( z \), \( j = i, e, h \). From (5), we have

\[ r = \alpha_i A_j k_i^{-\beta_i} - \delta_k, \quad w = \beta_i A_j k_i^{\alpha_j}. \]  

(A2)

From (A2) we determine \( r \) and \( w \) as functions of \( z \). From (5), (A2) and (A1), we solve

\[ p_j = \frac{\alpha_i \alpha_j^{\beta_j}}{\alpha_j^{\beta_j}}, \quad j = e, h. \]  

(A3)
From (A3) we determine $p_e$ and $p_h$ as functions of $z$. From (2) and (A1) we solve

$$\overline{\alpha}_i n_i + \overline{\alpha}_h n_h + \overline{\alpha}_e n_e = z k, \quad n_i + n_h + n_e = 1. \tag{A4}$$

Dividing (17) by $N_0$, we have

$$c + s - \delta \overline{k} = A_i n_i T H^m | m_h k_i^{\alpha_i}, \tag{A5}$$

where $\delta = 1 - \delta_k$. Substituting $c = \xi \overline{y}$ and $s = \lambda \overline{y}$ into the above equation yields

$$\overline{y} = \left( A_i n_i k_i^{\alpha_i} + \delta k \right) \frac{T H^m | m_h}{\xi + \lambda}, \tag{A6}$$

where we use $\overline{k} = k T H^m | m_h$. From (16) and (4), we have

$$\overline{\psi} \overline{y} = A_h k_h^{\alpha_h} p_h n_h T H^m | m_h, \tag{A7}$$

where we also use (3) and $p_h c_h = \overline{\psi} \overline{y}$. From (A6) and (A7), we solve

$$n_h = p_1 n_i + p_2 k, \tag{A8}$$

where

$$p_1(z, l, H) = \left( \frac{\overline{\psi}}{\xi + \lambda} \right) A_i k_i^{\alpha_i}, \quad p_2(z, l, H) = \left( \frac{\overline{\psi}}{\xi + \lambda} \right) A_h k_h^{\alpha_h} p_h.$$

Insert (A8) in (A4)

$$(\overline{\alpha}_i + \overline{\alpha}_h p_1) n_i + \overline{\alpha}_h n_e = z k - \overline{\alpha}_h p_2 k, \quad (1 + p_1) n_i + n_e = 1 - p_2 k. \tag{A9}$$

Solve (A9)

$$n_i = \Delta_i k - \overline{\alpha}_e \Delta, \quad n_e = \Delta_e - \Delta_0 k, \tag{A10}$$

where

$$\Delta_i(z, l, H) = (z - \overline{\alpha}_h p_2 + \overline{\alpha}_e p_2) \Delta, \quad \Delta_0(z, l, H) = [(1 + p_1) z + (\overline{\alpha}_i + \overline{\alpha}_h p_1) p_2 - \overline{\alpha}_h p_2 (1 + p_1)] \Delta, \quad \Delta_e(z, l, H) = (\overline{\alpha}_i + \overline{\alpha}_h p_1) \Delta, \quad \Delta(z, l, H) = \frac{1}{\overline{\alpha}_i + \overline{\alpha}_h p_1 - \overline{\alpha}_e (1 + p_1)}.$$
From (A10), we determine the labor distribution as functions of \( k, l, z \) and \( H \).

From (15) and (4), we have

\[
T_e = A_e n_e T H^m \left\| k_e^{\alpha_e} \right. .
\]

From (13) we have

\[
T_h = \frac{\bar{\psi} \bar{p}_e T_e}{\eta \bar{p}_h}.
\]

Insert (A11) and (A12) in \( T + T_h + T_e = T_0 \)

\[
T = T_0 \left[ 1 + \left( \frac{\bar{\psi} \bar{p}_e}{\eta \bar{p}_h} + 1 \right) A_e n_e H^m \left\| k_e^{\alpha_e} \right. \right]^{-1}.
\]

From the definition of \( \bar{\psi} \) and \( \bar{k} = k T H^m \left\| m_h \right. \), we have

\[
\bar{\psi} = (1 + r) k T H^m \left\| m_p + T_0 H^m \left\| m_h \right. w.
\]

Substituting (A14) into (A6) yields

\[
T_0 w = \frac{A_i n_i k_i^{\alpha_i} + \delta k}{\xi + \lambda} - (1 + r)k T.
\]

Substituting (A13) into (A15) yields

\[
w + \tilde{\Delta} n_e = \frac{A_i n_i k_i^{\alpha_i} + \delta k}{\xi + \lambda} - (1 + r) k,
\]

where

\[
\tilde{\Delta}(z, l, H) = \left( \frac{\bar{\psi} \bar{p}_e}{\eta \bar{p}_h} + 1 \right) A_e w H^m \left\| m_h \right. k_e^{\alpha_e}
\]

Insert (A10) in (A16)

\[
k(z, l, H) = \left( w + \tilde{\Delta} \Delta_e + \bar{\alpha}_e A_i k_i^{\alpha_i} \Delta \right) \left[ \frac{\Delta_i A_i k_i^{\alpha_i} + \delta}{\xi + \lambda} + \tilde{\Delta} \Delta_0 - (1 + r) \right]^{-1}.
\]
By (A17), we determine $k$ as functions of $l$, $z$ and $H$ at any point of time. By the following procedure, we determine the other variables as functions of $l$, $z$ and $H$: $k_j$ by (A1) → $r$ and $w$ by (A2) → $p_e$ and $p_h$ by (A3) $T$ $T$ by (A13) → $\bar{y}$ by (A14) → $\bar{k} = kT^Hm^l$ $m_k$ → $c$, $s$, $c_h$, $T_h$, and $T_e$ by (13) → $n_i$ and $n_e$ by (A10) → $n_h$ by (A8) → $\bar{c}$ by (8) → $K = \bar{k}N_0$ → $N$ by (3) → $N_j = n_jN$ → $K_j = k_jN_j$ → $F_j$ by (4).

We now express dynamics of the system in terms of $l(t)$, $z(t)$ and $H(t)$ at any point of time. First, from (18), (19), and the procedure described above, we have

$$\Phi(t) = \Omega_H(z, H, l),$$

$$\Phi(t) = \Omega_l(z, H, l),$$

(A18)

where $\Omega_H$ and $\Omega_l$ are functions of $l(t)$, $z(t)$ and $H(t)$. We do not provide the expressions of the functions as it is straightforward to calculate them but the expressions are tedious.

We express $\bar{k} = \Omega(z, H, l)$, where $\Omega$ is a function of $l(t)$, $z(t)$ and $H(t)$. We already described how to get $\Omega$ by the procedure. By (14), we have

$$\Phi = \Omega(z, H, l) = \lambda \bar{y}(z, H, l) - \Omega(z, H, l).$$

(A19)

Taking derivatives of $\bar{k} = \Omega(z, H, l)$ with respect to time, we have

$$\frac{\partial \Omega}{\partial z} \Phi + \Omega_H \frac{\partial \Omega}{\partial H} + \Omega_l \frac{\partial \Omega}{\partial l},$$

(A20)

where we use (A18). From (A19) and (A20), we have

$$\Phi = \Omega(z, H, l) = \left(\Omega - \Omega_H \frac{\partial \Omega}{\partial H} - \Omega_l \frac{\partial \Omega}{\partial l}\right) \left(\frac{\partial \Omega}{\partial z}\right)^{-1}.$$


