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Shear Stress Distributions along the Cross Section in Smooth Open Channel Flow

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ABSTRACT

Distributions of shear stress throughout the entire cross-section within the fully developed boundary layer region in a smooth and rectangular open channel flow setup are experimentally determined with the help of a laser Doppler anemometer. Velocity measurements are taken for 24 different sub-critical flow conditions with Froude numbers in the range: $0.13 \leq Fr \leq 0.78$ and with aspect ratios within $1.5 \leq B/H \leq 12.0$. For all the flow conditions created in the setup, the shear stress distribution is found to exhibit variations as a function of the aspect ratio, B/H, which is caused by the flow cells originated by the secondary flows in opposite direction to the main flow. For the aspect ratio between 1.5 and 5.0, the shear stress distributions are represented by a third degree polynomial function along the cross section. In this case, the maximum shear stress is 12% greater than the average stress. For $B/H \geq 5.0$, the dimensionless shear stress distributions are represented by a second degree polynomial. Here, the maximum shear stress was 10% greater than the overall average shear stress throughout the entire cross-section.

Keywords: Open channel flow, Shear stress distributions, Boundary layer

Introduction

Determination of shear stress distributions over a cross-section of an open channel is one of the basic problems in the area of hydraulics. Information regarding the boundary shear stress distribution in a flowing stream needs to be considered for various purposes such as giving a basic understanding of the resistance relationship understanding the mechanism of sediment transport and designing revetments for channels where meandering phenomena are predominant [1].

The boundary shear stress distribution and flow resistance in both compound and singular cross-section channels with smooth and rough surfaces have been investigated by many scientists [2, 3, 4, 5, 6, 7 and 8].

Knight et al. [9] presented equations to quantify the mean and maximum boundary shear stresses acting on the bed, τ_b, or on the walls, τ_w, of straight prismatic trapezoidal channels. They notified that no attempt was made to specify the lateral distributions of τ_b about the mean values due to the complex influence of secondary flow cells on τ_b.

In this study, many point velocity measurements were taken at high precision using a laser Doppler anemometer (LDA) throughout an entire cross-section within the fully developed boundary layer for 24 different sub critical flows in an open channel model. The shear stress right at the bottom where water is in contact with the channel bed was computed by the Newton shear stress law applicable in the viscous sublayer. The shear stress distribution was investigated along the cross section as a function of both the aspect ratio and the Froude number. Mean and maximum shear stresses and their positions along the cross section were determined for all flow conditions.

EXPERIMENTS

All the experiments were performed on the flume setup having a rectangular channel 30 cm wide, 40 cm deep, and 10 m long [10]. The flow rate of the water passing through the flume was measured with the help of a UFM-600 type of an ultra-sound current meter mounted on the pipe transferring the water from a constant-head tank to the flume entrance. The point velocities were precisely measured by a DISA-55-L type of a laser Doppler anemometer (LDA) which was installed on a special frame able to move in any direction.

The measurements were taken for 24 different uniform flows obtained out of five different flow rates. Pertinent information about the different types of flows produced in that particular setup are summarized in Table 1, where Q is the flow rate, B/H is the aspect ratio, Fr is the Froude Number $\left(=V/\sqrt{gH}\right)$, V is the average cross-sectional velocity, and Re is the Reynold Number (= 4VR/υ), in which R is the hydraulic radius, defined as: R=A/P, where A is the cross-sectional area and P is the wetted perimeter.

Development of the boundary layer in such an open channel flow was also investigated by Kırkgöz and Ardıçlıoğlu, [11]. The dimensionless length of the flow developing zone, at the end of which the boundary layer was fully developed, was found by the following simple expression:

$$\frac{L}{H}=76-0.0001\frac{Re}{Fr} \tag{1}$$

In all these investigated flow types, the section where the full boundary layer development was completed was located between 1.8 m and 6.5 m downstream from the channel entrance. Therefore, the section where the velocity and shear stress distributions were measured was consistently kept as the one 6.5 m downstream from the entrance for all types of flows studied. Point velocity measurements were taken by the mentioned LDA at various points on 8 vertical lines which were located at 2 cm adjacent distances from each other as shown in Figure 1. Due to the symmetry, the velocity measurements were taken only on one side of the cross-section.

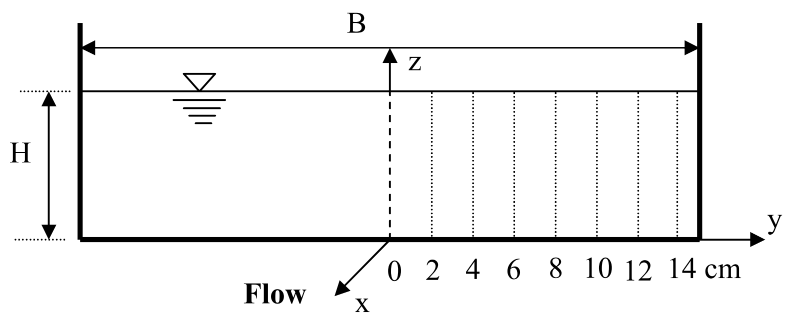

Figure 1 The vertical lines over the cross-section on which velocity measurements were made

Table 1 Description of the data used in the study

Test (1)	Discharge Q-(m^3/s) (2)	Slope S (3)	Water depth H-(mm) (4)	Aspect ratio B/H (5)	Fr Number (6)	Re Number (7)
1	0.0195	0.0005	200	1.5	0.24	101353
2	0.0195	0.0005	150	2.0	0.38	120526
3	0.0195	0.0005	120	2.5	0.55	139415
4	0.0195	0.002	100	3.0	0.69	142737
5	0.0145	0.0005	200	1.5	0.17	73684
6	0.0145	0.0005	150	2.0	0.27	86316
7	0.0145	0.0005	120	2.5	0.38	97544
8	0.0145	0.0005	100	3.0	0.52	108842
9	0.0145	0.002	80	3.8	0.78	126133
10	0.01	0.0005	200	1.5	0.13	55940
11	0.01	0.0005	150	2.0	0.21	67632
12	0.01	0.0005	120	2.5	0.29	74386
13	0.01	0.0005	100	3.0	0.36	74316
14	0.01	0.0005	75	4.0	0.55	83158
15	0.01	0.002	60	5.0	0.75	86767
16	0.006	0.0005	120	2.5	0.13	33684
17	0.006	0.0005	100	3.0	0.18	37684
18	0.006	0.0005	75	4.0	0.37	55789
19	0.006	0.0005	60	5.0	0.56	64662
20	0.006	0.002	45	6.7	0.71	57449
21	0.0032	0.0005	50	6.0	0.24	22368
22	0.0032	0.0005	40	7.5	0.36	24709
23	0.0032	0.0005	30	10.0	0.58	27456
24	0.0032	0.002	25	12.0	0.75	28045

RESULT and ANALYSIS

The shear stress occurring on the bottom of the channel was calculated using the Newton's shear stress equation, valid for the viscous sub layer, which is given as:

$$\tau_0 = \mu \frac{\partial u}{\partial z} \tag{2}$$

where, μ is the dynamic viscosity and u is the local velocity at the point that is z units away from the solid bottom. In the experiments it was possible to measure the flow velocity as near to the smooth bed as 0.3 mm. The thickness of the viscous sub layer in which the velocity distribution was linear varied from 0.3 mm up to 1 mm depending on flow conditions.

Accordingly, for all the 24 different types of flows, a sufficient number of point velocity measurements were taken within the viscous sub layer; and the linear velocity distributions were realistically determined over each vertical line transversely dispersed along the cross-section. A typical measured velocity profile on one such line, which is arbitrarily the middle-most line, and the shear stress computations at the bottom are shown in Figure 2.

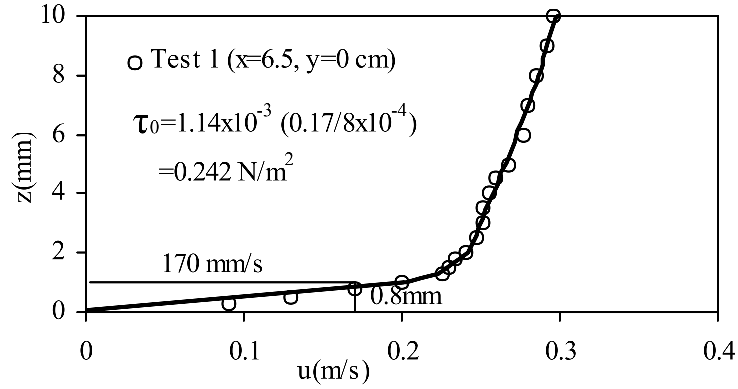

Figure 2 Velocity profile near the channel bed

Using the linear part of such velocity distributions valid in the viscous sub layer, the bottom shear stresses were calculated for all the eight vertical lines with all the 24 different flow types. The bottom dimensionless shear stresses in addition to the cross-sectional mean and maximum shear stresses are summarized in Table 2.

Table 2 Dimensionless shear stress

Test	Dimensionless shear stress τ_0/τ_m									
	2y/B								τ_m	τ_{max}
No	0	0.133	0.267	0.4	0.533	0.667	0.8	0.93		
1	0.954	1.095	1.067	1.112	0.988	1.011	0.988	0.786	0.254	0.282
2	1.148	1.062	1.038	1.017	1.058	1.070	0.989	0.619	0.350	0.402
3	1.140	1.174	1.070	1.125	1.003	0.958	0.891	0.639	0.384	0.450
4	1.149	1.101	1.079	1.162	1.059	0.957	0.830	0.664	0.447	0.519
5	0.980	1.063	1.013	1.096	0.980	1.005	1.038	0.824	0.173	0.190
6	1.113	1.045	1.113	0.950	1.018	1.106	1.018	0.638	0.210	0.234
7	1.065	1.117	1.158	1.065	1.088	1.054	0.845	0.608	0.246	0.285
8	1.083	1.011	1.028	1.105	1.149	1.160	0.856	0.608	0.258	0.299
9	1.157	1.107	1.101	1.101	1.016	0.994	0.960	0.565	0.505	0.584
10	0.978	1.043	1.033	1.065	1.011	1.011	0.989	0.870	0.131	0.140
11	1.125	1.068	1.035	1.002	1.018	1.035	0.977	0.739	0.173	0.195
12	1.042	1.034	1.034	1.017	1.051	1.017	1.000	0.805	0.168	0.177
13	1.017	1.034	1.017	0.988	1.114	1.091	1.051	0.689	0.248	0.276
14	1.028	1.040	1.080	1.126	1.040	1.097	1.011	0.578	0.247	0.278
15	1.095	1.071	1.047	1.053	1.041	1.023	0.918	0.752	0.474	0.519
16	1.062	1.062	1.043	1.025	1.007	1.025	1.007	0.769	0.078	0.083
17	1.063	1.050	1.037	1.050	1.050	1.037	1.010	0.704	0.107	0.114
18	1.175	1.104	1.110	1.050	1.014	0.996	0.847	0.704	0.239	0.281
19	1.041	1.053	1.061	1.049	1.061	0.939	0.914	0.882	0.349	0.371
20	1.148	1.169	1.148	1.107	0.976	0.902	0.820	0.730	0.348	0.406
21	1.021	1.021	1.021	1.008	1.021	0.982	0.982	0.944	0.112	0.114
22	1.049	1.040	1.067	1.012	1.021	0.994	0.949	0.868	0.158	0.168
23	1.183	1.171	1.117	1.123	0.918	0.906	0.894	0.688	0.236	0.279
24	1.100	1.082	1.096	1.037	1.028	0.974	0.938	0.744	0.316	0.348

It is a known fact that the gravitational forces play a dominant role on the movement of open channel flow as rephrased by Chow also in his statements: "The effect of gravity upon the state of flow is represented by a ratio of inertial forces to gravity forces. This ratio is given by the Froude number", and further: "It is believed that gravity action may have a definitive effect upon the flow resistance in channels at the turbulent flow range." [12]. As shown in Figure 3, the cross-sectional mean shear stresses, τ_m, for 24 flow conditions increased linearly versus Froude numbers. The relation between cross sectional mean shear stresses and Froude numbers are determined by the following expression:

$$\tau_m = 0.475\,\mathrm{Fr} + 0.057 \tag{3}$$

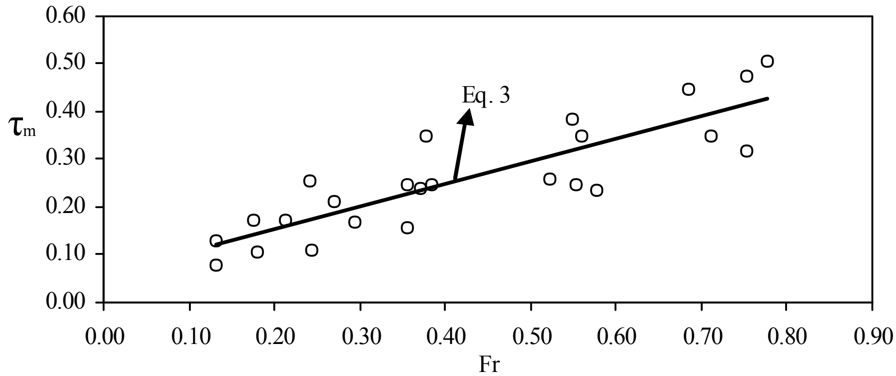

Figure 3 Relation between cross sectional mean shear stresses and Froude numbers.

The determination coefficient of the best fit regression equation is: R^2= 0.74. During the efforts of determining the most meaningful expressions where the average shear stress was the dependent variable, all the other possible combinations in which the Reynold number, Re, and the aspect ratio, B/H, appeared as prospective independent variables were also tried, but these expressions revealed insignificant effects of both Re and B/H on average shear stress, and therefore, the average shear stress appeared as a function of the Froude number only.

For all the flow conditions formed in the setup, the shear stress distribution along the cross section was found to exhibit variations as a function of the aspect ratio, B/H. Dimensionless shear stress, τ_0 / τ_m, distributions for $1.5 \leq B/H < 5.0$ were found to be close to each other. Figure 4 shows these variations along the cross section for fully developed flow conditions. In the figure dimensionless shear stress distribution shows an oscillatory behavior which was explained by the positioning of the secondary current cells created by the corner boundaries in straight channels. This distribution can be determined as a third degree polynomial as shown in Figure 4. The regression equation fitted to this relation, whose determination coefficient is: R^2= 0.78, is:

$$\frac{\tau_0}{\tau_m} = -2.05(2y/B)^3 + 2.06(2y/B)^2 - 0.55(2y/B) + 1.09 \tag{4}$$

The maximum shear stress on the bed has been found to be about 12% greater than the mean bed shear stress when $1.5 \leq B/H < 5.0$ throughout the entire cross-section. The maximum shear stress does not occur at the center line for the aspect ratios between 1.5 and 5.0. As getting closer to the side wall, the shear stresses consistently decreased, which were due to the effect of the flow cells induced by the secondary flows in opposite direction to the main flow.

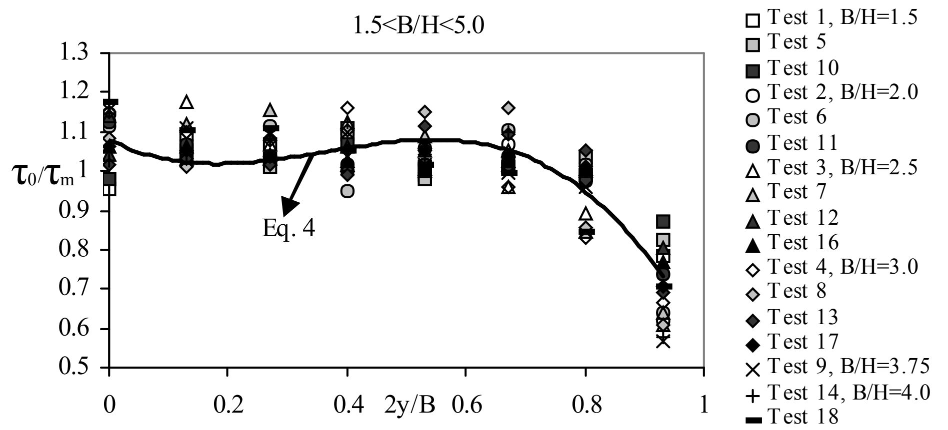

Figure 4 Variations of τ_0 / τ_m along the cross section for 1.5≤ B/H <5.0

Knight and Patel (1985) reported that the maximum bottom shear stress was 13% greater than the average value. Dimensionless shear stress distributions, τ_0 / τ_m, show second degree polynomial when B/H≥ 5.0 as given in Figure 5. In this case, the maximum shear stress occurs mostly at the middle section of the channel and decreases close to the side wall. The regression equation fitted to this relation, whose determination coefficient is R^2= 0.75 is:

$$\frac{\tau_0}{\tau_m} = -0.41\,(2y/B)^2 + 0.084(2y/B) + 1.087 \tag{5}$$

In these cases, the maximum shear stress in the middle-most vertical line was 10% greater than the overall average shear stress throughout the entire cross-section.

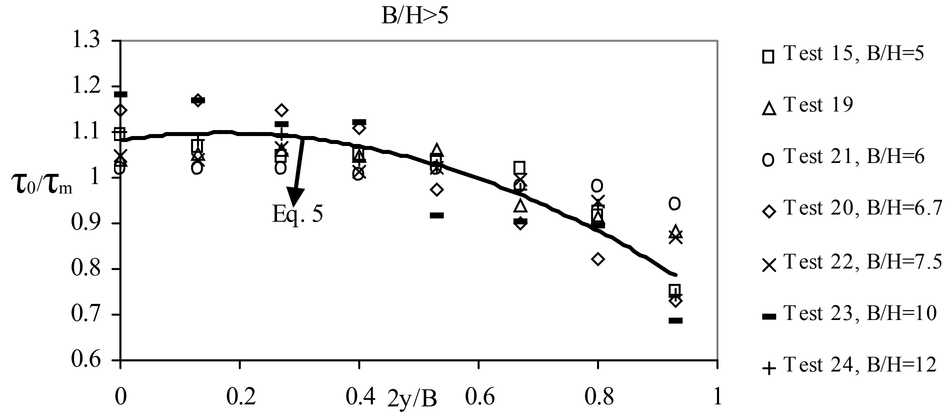

Figure 5 Variations of τ_0 / τ_m along the cross section for B/H≥ 5.0

CONCLUSIONS

Boundary shear stress distributions have been measured in fully developed turbulent flows in a smooth rectangular open channel with aspect ratios between 1.5 and 12. The shear stress distributions were calculated using the linear part of such velocity distributions valid in the viscous sublayer for eight vertical lines along the cross-section. The cross-sectional mean shear stresses were determined to increase linearly with the Froude number. The relation between the cross-sectional mean shear stresses and the Froude number is represented by a statistically significant expression. The shear stress distribution along the cross-section exhibited two fairly distinct variations depending on two intervals of the magnitudes of the aspect ratio, B/H. For the interval of: $1.5 \leq B/H < 5.0$, the dimensionless shear stress distributions, τ_0 / τ_m, is represented by a third degree polynomial. The maximum shear stress on the bed has been found to be about 12% greater than the mean bed shear stress for: $1.5 \leq B/H < 5.0$ throughout the entire cross-section, and the maximum shear stress does not occur at the center line for this interval of B/H. τ_0 / τ_m is accurately represented by a second degree polynomial when $B/H \geq 5.0$. In this case, the maximum shear stress occurs mostly in the middle section of the channel. And, the maximum shear stress is found to be about 10% greater than the overall average shear stress throughout the entire cross-section.

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