Static Design of Systems with Semi-Rigid Connections Based on Experimental Investigation of the Full Scale Structure

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ABSTRACT

The paper presents the basic theory and corresponding equations for calculation of the static influences of the structures with semi-rigid connections in joints. The static design of system in which connections of members are absolutely rigid or perfectly pinned has been thoroughly worked out in the existing literature. In real structures in general, and particularly in the precast ones, connection in joints may be partially rigid, which can have a significant impact on the change of stresses and strains in the structure. In the world there are several procedures, with different approaches, to the structural design with semi-rigid connections, however, it is proved that they all lead to the same results. All of these approaches in addition to theoretical deal with experimental analysis of results. According to the approach that will be presented in this paper, typical prefabricated structures "Minoma" 1, "Minoma" 2, and "Minoma" 3 with span of 12m, 20m and 27m respectively, and the height of the column up to 9m, have been experimentally analyzed. A numerical example will be given and discussed, where bending moments due to a given load, as well as due to input force in the tensioner, have been calculated for defined levels of rigidity of connection using the deformation method. The obtained superimposed bending moment diagram will be analyzed too.

Keywords: Precast reinforced concrete structure, full scale test, semi-rigid connection, deformation method.

INTRODUCTION

Design of the systems is most often conducted with the assumption that the members are connected in nodes in an either completely rigid (rigid angle) or ideal joint manner. In the actual structures and especially in pre-fabricated ones, these connections, that is, joints are as a rule semi-rigid (elastic) which has a significant impact on the redistribution of stress and strain in the structure of the building. Creation of a durable connection between the elements of pre-fabricated systems is done in several ways: according to the dry mounting method, wet procedure or by pre-stressing, depending on the structure. As for the concrete pre-fabricated structures, apart from the quality designing, calculation and construction, connection of pre-fabricated elements must be paid a special attention. Connections are between the elements and result in monolithic character of a structure and in the static and dynamic stability, thus their role is multiple. Depending on the function which ought to be performed, the

connections can differ in terms of: purpose, form, position and function of the elements they connect, forces they must receive and transfer, external elements and applied materials, construction technology and other.

Calculation of linear systems, assuming that the members in the nodes are connected by rigid or joint connection, is known as approximate strain method, which will not be presented here as it is generally known, can in the similar way be applied to the systems with semi-rigid – elastic connections, only with the application of appropriate values of semi-rigid connections, as it is given in the references [1].

Apart from the analytical a series of experimental static and dynamic researches have been conducted. The modern approach of analytical calculation allowed complete analysis of simulated behavior of structures in the most adverse loading conditions. However, accuracy of certain parameters of mathematical models (static and dynamic) was experimentally verified on the full scale structures in order to determine the safety of these structures.

HISTORICAL DEVELOPMENT OF DESIGN OF STRUCTURES WITH SEMI-RIGID CONNECTIONS

As early as in 1917 Wilson, Moor and Batho conducted first experimental research on steel structure in order to define the analytical relation between the moment of rotation in riveted and welded connections and point to the differences in behavior of these two types of connections.

The similar tests were performed in 1930 by Rowan and Rathbun. Only as late as in 1963, on the basis of the results of previously conducted research in this field, Monforton and Wu defined the relation between the moment and rotation as a linear with the proportionality coefficient which represents the rigidity of the connection. On the basis of such relation, the same authors derived the stiffness matrix for the elements with semi-rigid joins. The Russian scientist Denkevich studied reinforced-concrete frames with "yieldable" nodes in 1967. Shapiro publishes a paper in 1974 about the effects of yieldability of nodes of reinforced-concrete frames at horizontal load, and this problem was treated by M. Milićević, S. Zdravković and R. Folić.

In the previous thirty years a large number of papers in the field of semi-rigid joints was published in the world. Many authors proposed the methods with whose aid the influences of such connections on the global analysis of the structure can be included. The obtained results mostly coincide and correspond to the real behavior of the structure. The research field was expanded from the elastic to plastic area, so in a global plastic analysis of semi-rigid joint they can be approximated with their generally non-linear characteristics.

There are symposiums every alternate year, as opportunities to explain different views and to meet one another (Strasbourg 1992, Prague 1994, Istanbul 1996) and to exchange experiences acquired in earlier researches, and the results are printed in dedicated publications. In the period 1991 to 1999 a wider European program, project COST C1 „Control of the semi-rigid behavior of civil engineering structural connections" was established. The most important results of the COST C1 project have been used for practical methods, standards, codes and design procedures in the corresponding field of similar materials. Some general results were achieved in the area of frame analysis irrespective of the material. The majority of researchers is focused on the experimental work. Most of the experiments are conducted by testing the substructures.

STATIC DESIGN OF THE STRUCTURES WITH SEMI-RIGID CONNECTIONS

The structures where the mutual member connections are not absolutely rigid (rigid angle) but permit a certain relative rotation of extreme cross sections in the nodes are the

systems with semi-rigid connections of members in nodes. As such connection system is considerably frequent in structures, particularly in the pre-fabrication ones, it is interesting to consider their design taking into account the elasticity of nodal connections.

The expressions for moments at the ends of members have been derived for the semi-rigidly connected members in nodes, as well as conditional equations of the strain method. If the designations $\mu_{ik} = \varphi_{ik}^* / \varphi_i \mu_{ki} = \varphi_{ki}^* / \varphi_k$, are introduced, where φ_1 and φ_k are rotation angles of the node i i.e. k, and φ_{ik}^* and φ_{ki}^* are rotation angles of extreme cross sections of the member ik and they are named the degrees of fixation of the member ik in the nodes i and k, then, the expressions for the moments at the ends of the members fixed in such manner will be:

$$M_{ik} = a_{ik}\varphi_i^* + b_{ik}\varphi_k^* - c_{ik}\psi_{ik} + m_{ik}^{(0)} + m_{ik}^{(\Delta t)} \tag{1a}$$

$$M_{ki} = b_{ik}\varphi_i^* + a_{ki}\varphi_k^* - c_{ki}\psi_{ik} + m_{ki}^{(0)} + m_{ki}^{(\Delta t)} \tag{1b}$$

Or via the rotation angles of the nodes φ_i, φ_k

$$M_{ik}^* = a_{ik}^*\varphi_i + b_{ik}^*\varphi_k - c_{ik}^*\psi_{ik} + m_{ik}^{(0)^*} + m_{ik}^{(\Delta t)^*} \tag{2a}$$

$$M_{ki}^* = b_{ik}^*\varphi_i + a_{ki}^*\varphi_k - c_{ki}^*\psi_{ik} + m_{ki}^{(0)^*} + m_{ki}^{(\Delta t)^*} \tag{2b}$$

The constants $a_{ik}^*, b_{ik}^*, c_{ik}^*$, as well as the initial moments of semi-rigidly fixed members can be expressed through the corresponding values of rigidly fixed member (the designations without *) and degree of fixation in the following way:

$$a_{ik}^* = \mu_{ik}\left[a_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}b_{ik}\right], \qquad a_{ki}^* = \mu_{ik}\left[a_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}b_{ik}\right], \qquad b_{ik}^* = b_{ki}^* = \mu_{ik}\mu_{ki}b_{ik},$$

$$c_{ik}^* = \mu_{ik}\left[c_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}c_{ik}\right], \qquad c_{ki}^* = \mu_{ki}\left[c_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}c_{ik}\right], \tag{3}$$

$$m_{ik}^* = \mu_{ik}\left[m_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}m_{ki}\right], \qquad m_{ki}^* = \mu_{ki}\left[m_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}m_{ik}\right], \tag{4}$$

$$M_{ik}^* = \mu_{ik}\left[M_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}M_{ki}\right], \qquad M_{ki}^* = \mu_{ki}\left[M_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}M_{ik}\right]. \tag{5}$$
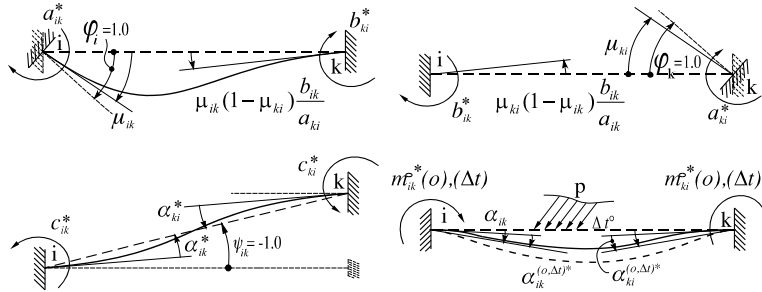

Figure 1. Physical meaning of the constants of members and initial moments of semi-rigidly fixed member

And their physical meaning is presented in the figure 1, where:

$$\alpha_{ik}^* = \mu_{ik} - (1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}}, \qquad\qquad \alpha_{ki}^* = \mu_{ki} - (1-\mu_{ki})\mu_{ik}\frac{b_{ik}}{a_{ki}}$$

$$\alpha_{ik}^{*(0,\Delta t)} = \mu_{ik}\alpha_{ik}^{(0,\Delta t)} - (1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}}\alpha_{ik}^{(0,\Delta t)}, \qquad \alpha_{ki}^{*(0,\Delta t)} = \mu_{ki}\alpha_{ki}^{(0,\Delta t)} - (1-\mu_{ki})\mu_{ik}\frac{b_{ik}}{a_{ki}}\alpha_{ki}^{(0,\Delta t)} \tag{6}$$

are obtained through the superposition principle .

In the expressions (3) it can be observed that by varying the values μ_{ik} and μ_{ki} with 1 and 0 the members according to the first order theory are obtained : type k ($\mu_{ik} = \mu_{ki} = 1$), type g ($\mu_{ik} = 1$; $\mu_{ki} = 0$) and type s ($\mu_{ik} = \mu_{ki} = 0$), so in the further analysis all can be treated as a single type of semi-rigid fixed member, which significantly simplifies and uniforms the design, which is particularly important for making of the computer software for structural calculation. The calculation is performed by the classic strain method including the semi-rigid connections of members in the nodes. For calculation of the values given in the expressions (4) and (5) for the members with the constant cross-sections for the various degrees of fixation μ_{ik} and μ_{ki} the computer software MK-TAB [4] has been composed. The bending moments at the ends of the semi-rigidly fixed member M_{ik}^* and M_{ki}^* can be written in the following form:

$$M_{ik}^* = a_{ik}^* \varphi_i + b_{ik}^* \varphi_k - c_{ik}^* \sum_{j=1}^{n} \psi_{ik}^{(j)} \Delta_j + m_{ik}^* \tag{7a}$$

$$M_{ki}^* = b_{ik}^* \varphi_i + a_{ki}^* \varphi_k - c_{ki}^* \sum_{j=1}^{n} \psi_{ik}^{(j)} \Delta_j + m_{ki}^* \tag{7b}$$

Rotation and displacement equations are now:

$$\sum_k M_{ik}^* + M_i = 0 \qquad (i=1,2,...,m) \tag{8a}$$

$$\sum_k (M_{ik}^* + M_{ki}) \psi_{ik}^{(j)} + R_j = 0 \qquad (j=1,2,...,n) \tag{8b}$$

When the equations (7) are entered in the equations (8) and the expressions are arranged, the node rotation equations and displacement equations are obtained:

$$A_{ii}^* \varphi_i + \sum_k A_{ik}^* \varphi_k + \sum_{j=1}^{n} B_{ij}^* \Delta_j + A_{i0}^* = 0 \qquad (i=1,2,...,m) \tag{9a}$$

$$\sum_{i=1}^{m} B_{ji}^* \varphi_i + \sum_{l=1}^{n} C_{jl}^* \Delta_l + C_{j0}^* = 0 \qquad (j=1,2,...,n) \tag{9b}$$

Where the following designations have been introduced:

$$A_{ii}^* = \sum_k a_{ik}^* + \sum_c e_{is}^*, \qquad A_{ik}^* = B_{ik}^*, \qquad A_{i0}^* = \sum_k m_{ik}^* + M_i \tag{10a}$$

$$B_{ij}^* = -\sum_k c_{ik}^* \psi_{ik}^{(j)} = B_{jk}^* \tag{10b}$$

$$C_{jl}^* = \sum_{ik} \left(c_{ik}^* + c_{ki}^* \right) \psi_{ik}^{(j)} \psi_{ik}^l \mp EI_c \sum_{ab} \frac{\omega_{ab}^2}{L_{ab}} \psi_{ab}^{(j)} \psi_{ab}^{(l)} \tag{10c}$$

$$C_{j0}^* = -\sum_{ik} \left(m_{ik}^* + m_{ki}^* \right) \psi_{ik}^{(j)} - R_j \mp EI_c \sum_{ab} \frac{\omega_{ab}^2}{L_{ab}} \psi_{ab}^{(j)} \left(\psi_{ab,t} + \psi_{ab,c} \right) \tag{10d}$$

By comparing the expression (10) with the corresponding expressions for rigidly fixed members, it can be concluded that they are formally identical, only the constants of the members and the initial moments are determined according to the expressions (4), and in the expressions (8) and (9) , $\sum_{ik} and \sum_{ig}$ do not appear, because all the members are introduced in the first sums.

EXPERIMENTAL TESTS ON THE FULL SCALE STRUCTURES WITH SEMI-RIGID CONNECTIONS

General

In structural analysis, both material and mathematical (calculation) models can be used. The Material models require significant investments, but their usage is justified in the case of large atypical structures, or when a large series of equal structures is constructed, as is most often the case with pre-fabricated structures, so the behavior of an entire batch can be determined on one model.

Development of prefabricated systems has an increasing worldwide application nowadays. Industrial production of structural and non-structural elements in the factories achieves a consistent quality and cost-efficiency. Their assembling on-site, makes these systems progressively acceptable for typical structures such as: industrial halls, farms, airport and other warehouses etc.

The basic requirements in development of pre-fabricated systems are that the prefab structure is: more cost-effective in comparison to the classic one, simpler, easy for transport and enables fast and easy assembling. These requirements can be met only with the aid long term and extensive analytic and experimental static and dynamic tests.

It is well known that in the present time, contemporary principles of analytic calculations enable correct simulation of structural behavior in the most adverse loading conditions. Accuracy of certain parameters of the mathematical model must be verified through experiments. Prefabrication construction necessitates study of connecting means and method of connection of prefabrication elements into a single structure.

On the basis of theoretical and experimental research at the Faculty of Civil Engineering and Architecture of Nis, (Serbia) a new standardized prefabricated reinforced concrete structure „MINOMA" was designed, for structural spans from 9 to 27m and of the height of 7,5m (9m) with the span of main beams from 3 to 6m. The purpose of the structure has been primarily keeping fowl, kettle, pigs, sheep etc.

Structural properties

The main support structure of MINOMA is composed of straight prefabricated members which are rigidly connected in nodes by steel or aluminium connectors. In the support points, the columns are fixed in mounting footings, whereas they are connected with braces at the tops. In this way a standard frame is formed which in the static terms represents a frame structure with semi-rigid connections of members and braces mounted at the level of column heads. (Fig.3.)

The geometrical system is the same for all the types of MINOMA structure. The cross sections of columns and beams, installed reinforcement, footings and connection details can be observed. The columns and beams in the MINOMA-1 structure have the cross section
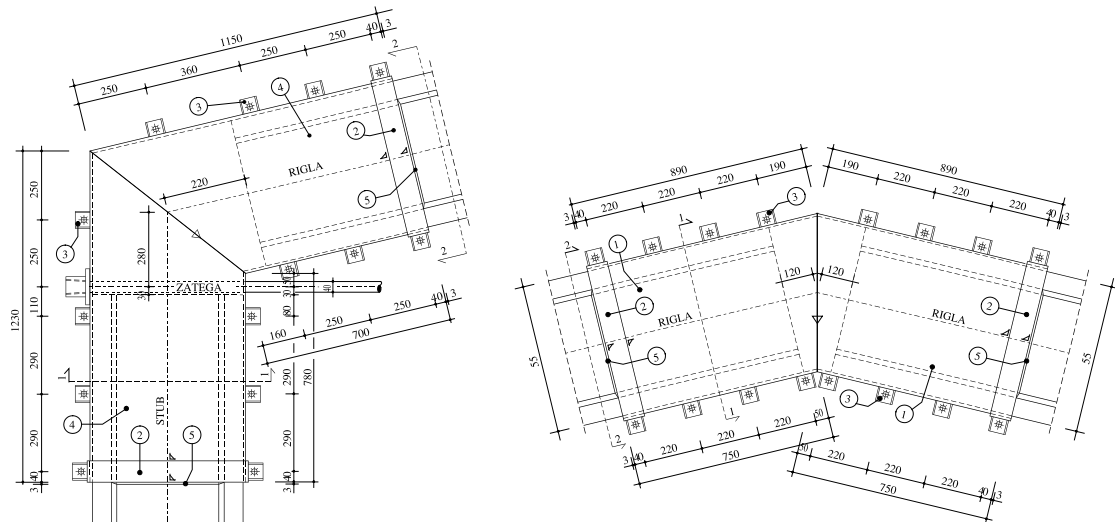
Figure 2. Nodal connections of „MINOMA" frames

20/22cm, and in the MINOMA-2 25/35cm and in the MINOMA-3 35/55cm. In the Figure 3. the cross sections of these elements are given, and the concrete grade is MB45 and Č180, reinforced with 11Ø3, 15Ø3 and 25Ø3 respectively, with haunches Ø6/15/7,5. The testing determined the maximum spans of the type MINOMA structures: for MINOMA-1 L≤13,4m, for MINOMA-2 L≤20,4m and for MINOMA-3 L≤27,2m. The maximum height is H≤7,5m (9m), and the distance of main beams (λ) is 3 to 6m. The brace is of high-grade steel of the IMS system of galvanized wire (cables 6Ø7mm or 12Ø5mm). In this way a standard frame is formed, which in the static terms is a frame structure with steel member connections and with brace connected with joints.

DETERMINATION OF STATIC CHARACTERISTICS OF STRUCTURES BY EXPERIMENTS ON THE FULL SCALE MODEL AND CALCULATION MODEL

Static testing

These tests are based on the JUS U.M1.047 regulations. The tests were performed in two stages:
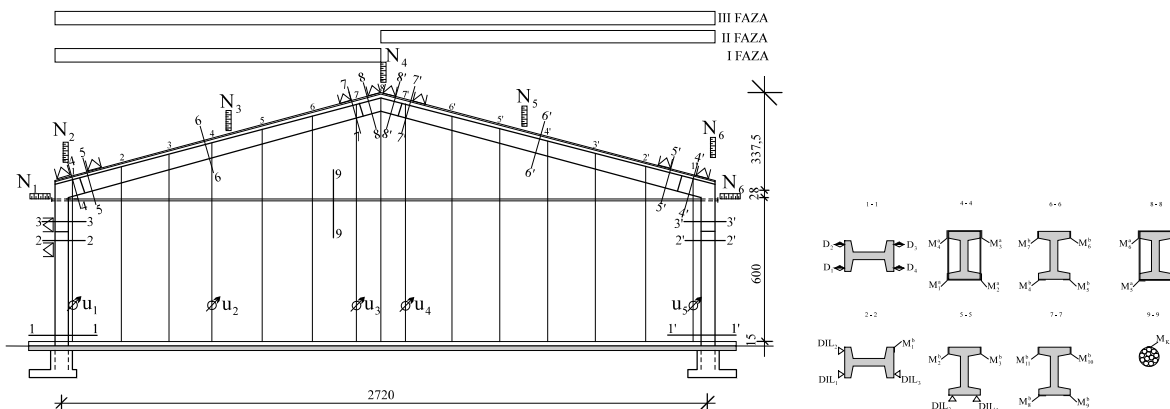

Figure 3. Designations and arrangement of measuring cross-sections for static tests of frame of MINOMA structure and its cros-sections

1) I stage: tests to the limit of elasticity (serviceability),
2) II stage: tests until failure (tests of individual elements and subsystems until failure).

Static tests were conducted in three phases according to Fig.3. The load was applied by the braces fixed at the reinforced concrete floor, and with counter-beams installed. The force in the braces was monitored by measuring tapes, with automatic compensation of temperature. The initial force in the brace amounted to Z=83kN.

The choice of measuring cross-sections was conducted on the basis of expected maximum static influences so that they could be monitored at test load.

Control of deflection by the kinematic method and tangent gradient – measured values
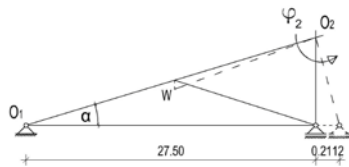
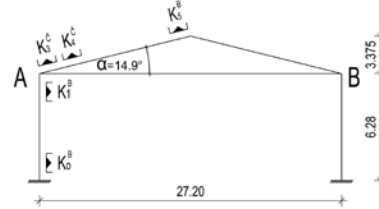Figure 4. Deflection at the ridge determined by the kinematic energy

Figure 5. Designation and arrangement of clinometers

Control of deflection by the kinematic energy, as presented in Fig. 5. is done according to the measured strains:

E=210 000 000N/m², $\alpha = 14,905°$, F=4,618cm², ε=77·10⁻⁶ measured strain

Z=77·10⁶·2,1·10⁶·4,618=7,467kN – force in the connection,

ΔL=ε·L=77·10⁶·27500=2,117mm

φ₂=2,117/7320=0,0002892=2,893·10⁻⁴, υ=2,893·10⁻⁴·14228=4,116mm,

w=4,116·0,9663=3,97mm, measured deflection is 4,1mm.

In the Fig.6. the necessary values for determination of the degree of fixation have been stated: $k_o^B = 46``$ , $k_1^B = 6``$, $k_3^B = 99``$ , $k_4^B = 46``$, $k_5^B = 7``$. For the degree of fixation of the beam and the column (A) from the previous measuring, the values of tangent are: $\alpha_3^{\check{c}} = 99``$

clinometer K^B, $\alpha_g^B = 46``$ clinometer K$^4$. It is $\dfrac{\alpha}{\alpha``} = \dfrac{2\pi}{360 \times 60 \times 60} = 4,85 \times 10^{-6}$,

$\alpha = \alpha`` \times 4,58 \times 10^{-6}$ , $\alpha_3^{\check{c}} = 99`` \times 4,85 \times 10^{-6} = 0,00048 rad$,

$\alpha_4^B = 46 \times 4,85 \times 10^{-6} = 0,00022 rad$, $(6``) = 0,000029 rad$ , $(7``) = 0,000034 rad$,

$\alpha_{ru\check{c}} = 0,00022 rad$ for the node 4. The similar is obtained for the node B. The degree of fixation for node 0: $\alpha_{ra\check{c}} = 0,00027 rad$, $\alpha_{mer}`` = k_0^2 = 46``$, $\alpha_{mer} = 46`` \times 4,85 \times 10^{-6} = 0,00022 rad$.

On the basis of measuring and calculation the degree of fixation of the nodes A and B is obtained, amounting to 80% to 100%, and for fixation with the soil from 40% to 50% depending on the type of MINOMA.

Analysis of static test results

For the construction and loading presented in the figure 4 and determined degrees of fixation by the strain method which is presented in detail in the section 3, the bending moment at the static load of the dead weight and applied load have been calculated according to the First order theory. Independently are calculated influences of evenly distributed loading of the intensity q=235daN/m and of the applied force in the bracing which is decreased for the determined value of the loss of 20% (S=0,8Z=66,4kN), after which the superposition was accomplished. The bending moment diagrams are calculated according to (7) and presented in Fig. 6.
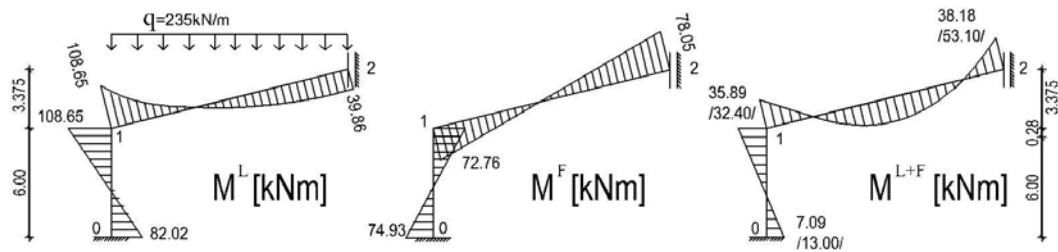
Figure 6. Bending moment diagrams due to: a) applied load, b) tie force, c) sum of a) and b)

ECONOMIC COMPARATIVE ASPECTS

Long lasting research [5] and [6] produced a system of standardized concrete pre-fabricated structures with the following properties: production in an industrial manner in throughout a year, very small weight of the elements, allowing easy transport and fast mounting on-site in all weather conditions. These characteristics are very important for the cost of the goods. These researches demonstrated that this structure is at least 30% more cost-effective than the similar, known structures, and in comparison to the classic frame structure, constructed on-site, even more, which means considerable saving in the total cost of the building price. Its versatility adds to its competitiveness.

CONCLUSION

The structures with semi-rigid joints are those systems where the connections of the members in nodes are not absolutely rigid, but permit, in the general case relative displacement in the directions of generalized displacement. In the present day engineering practice, in design of the structure, this fact has been marginally taken into consideration, if at all. If the influence of semi-rigid connections is underestimated and if they are treated as joints this has negative effects on the cost-efficiency of the structure. If an unrealistically high degree of fixation of members in nodes is assumed, the obtained results do not favor safety, which may have negative effects on bearing capacity, durability and stability of the structure.

It has been demonstrated that the existing software packages for static and dynamic calculation of structures can be applied to the calculation of structures with semi-rigid connections of members in nodes.

The conducted static research of a standardized system MINOMA indicate that the system behaves elastically and that there are no retained deflections and stresses, nor cracks. In the course of testing no anomalies were observed on the structure. The results of departure of individual parameters of the cross sections are certainly the influence of connection rigidity and their elastic behavior.

For the applied test load, the obtained global safety coefficient by deflections was 2,02.

For the tests until failure, the safety coefficient obtained was 3,8.

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