The assessment of the safety factor with Probability Based Design and Euro code 7 in a slope stability case study

Luisa Dhimitri[1], Ergys Anamali[1]

1 Department of Civil Engineering, Polytechnic University of Tirana, Albania

ABSTRACT

Slope stability accidents are widely spread in geotechnical calculation field.

The purpose of this paper is to present an artificial slope stability case study, constructed in Porto Romano area (near Durres city, Albania). This embankment was foreseen to be building in a conic trunk shape, with diameter in base 43 m, height 10 m and inclination ratio 1:3, which was also used as a load during the consolidation process within the Preloading Mitigation Seismic Risk study of this area.

The way of embankment's construction was suggested by the team of engineers, which performed the slope stability calculations during the design phase.

Our study aims to deal with the recalculation of safety factor using Probability Based Design and Eurocode 7 Design Approaches after it failed at 6.45 m of height.

After the recalculation we could estimate the causes that led to the failure.

Keywords: *safety factor, reliability, probability of destruction, partial factors.*

INTRODUCTION

We will report the results of a study of the failure of an artificial slope that had been constructed in the industrial area near Durresi city, as a part of the PREMISERI project for improving the characteristics of the ground. The embankment was build during the summer of 2011. This embankment was built in a truncated-cone-shaped. They have planned to use a 50 m diameter fill, 9 m high, 13 m diameter at the crest, as shown in fig. No.1.The embankment base location was specified by topographic means. The embankment was slowly constructed in 20 cm layers. The soil used to construct the embankment was **silty SAND**. The properties of the filling material were: $\gamma_1 = 21.15$ kN/m^3 and $\varphi = 34°$ (type SM). Compaction of the layers was performed with a steam roller. From the first calculation performed by the designers team the safety factor resulted $F_s = 2.30$. They took into consideration the first layer of soil (depth 0.0 until -3.5 m) during their calculation using the following properties: Type of soil: CH; Bulk density $\gamma = 14.0$ kN/m^3; Internal friction angel $\varphi = 20°$; Cohesion c= 20 kPa. The underground water level considered is 0.00 m. [1]

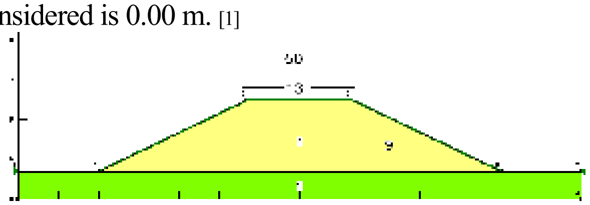

Fig. No.1 The initial slope

Failure of the slope occurred when the embankment reached at 6.35 m. The depth of the failure surface was until 5.35 m.

During the reconstruction they used the same material. After the failure, the bulk density of it changed into $\gamma_2 = 19.61$ kN/m^3. This amount of material was used to build the embankment from 1.00 m until 6.35 m. They continued building it from 6.35 m until 9.00 m using new material same with the material used for building it from 0.00 m until 1.00 m ($\gamma_1 = 21.15$ kN/m^3 and $\varphi = 34°$ (type SM)). A schematic illustration of the slide and a topographic imprint of the slides are given in fig. No. 2. [1]

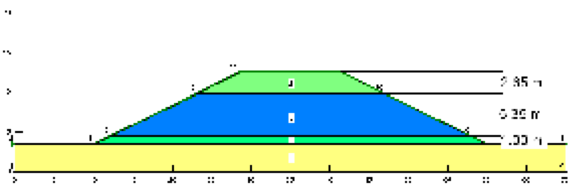

Fig. No.2 a) The slope after failure

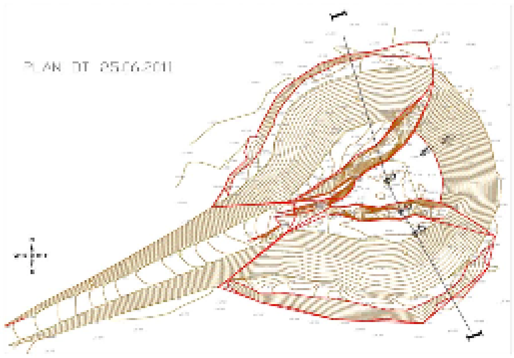

Fig. No.2 b) The topographic imprint of the slides

1. THEORICAL BACKGROUND OF THE BACK CALCULATION

Our back calculation is conducted in relation to first calculations (done from the designers of this embankment) and to Probability Based Design and Eurocode 7.

1.1. Slope Stability Analysis according to Probability Based Design

Reliability "R" in slope stability analysis can be defined as a way of measuring the stability that takes into account all the uncertainties involved in the process. It is equal to the probability that a slope will not fail and it can be calculated as follows:

$$R = 1 - P_f \tag{1}$$

Standard deviation and coefficients of variation. The change of values obtained from a test done to a sample of soil is due to natural variation, amount of sample disturbance etc. In order to define the scatter of a variable it is introduced the standard deviation which can be calculated using the formula below:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{1}^{N}\left(x - x_{av}\right)^2} \tag{2}$$

Where: σ is the standard deviation, N the number of measurements, x the measured variable and x_{av} the average value of x.
The coefficient of variation is a dimension of uncertainty in the value of the variable. It can be calculated by formula no. (3).

$$COV = \frac{\sigma}{\text{average value}} \tag{3}$$

COV of factor of safety and the reliability index

In order to calculate the reliability (R) and probability of failure (P_f) it is required to estimate firstly the factor of safety and the coefficient of variation of factor of safety (COV_F). The factor of safety can be calculated as usual, using a computer or by hand calculations. In this paper the factors of safety are retrieved from the commercial software GEOSLOPE SLOPE/W. After this, it can be estimated the value of the coefficient of variation of factor of safety using Taylor series method. To apply this method, first of all we need to estimate the standard deviations of all the quantities included in the slope stability process and then to use formula (5) and (6):

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \ldots + \left(\frac{\Delta F_n}{2}\right)^2} \tag{5}$$

$$COV_F = \frac{\sigma_F}{F_{MLV}} \tag{6}$$

Where: $\Delta F_n = \left(F_n^+ - F_n^-\right)$, F_n^+, F_n^- the value of the factor of safety calculated with the value of the n –th parameter plus/ minus one standard deviation from its most likely value. After these assessments the value of σ_F and COV_F can be easily retrieved. Using these two values the value of the probability of failure can be determined according to the graph represented in Fig. No. 3.
On the other hand the probability of failure P_f is strongly related to the *lognormal reliability index* (β) which is determined using the formula (7):

$$\beta_{LN} = \frac{\ln\left(F_{MLV} / \sqrt{1 + COV_F^2}\right)}{\sqrt{\ln\left(1 + COV_F^2\right)}} \tag{7}$$

Values of P_f that corresponds to the calculated β_{LN} values can be estimated using the "NORMSDIST" function in Microsoft Excel. The function returns the value of R, but using equation (1) we can obtain P_f. [2]

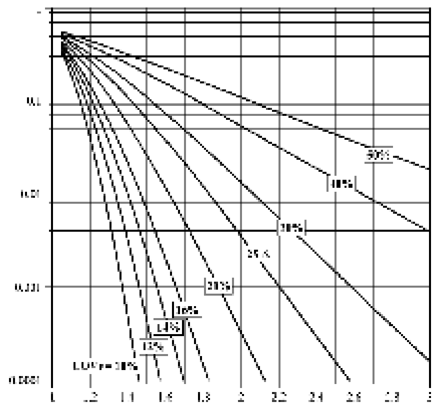

Fig. No. 3 Relation between FS and P_f

1.2 Slope Stability Analysis according to Eurocode 7

This analysis is done taking into consideration two limit states: (1) Ultimate limit states (ULS), including GEO and STR and (2) Serviceability limit states (SLS).

Ultimate limit state design

The stability analysis of a slope should be performed using the factored values of material properties. The design approaches and the partial factors for this purpose are stated in Eurocode 7. In our case it is convenient to use Design Approach 1, Combination 1 (which gives the same results as Design Approach 3) and Design Approach 1, Combination 2 (which gives the same results as Design Approach 2). The recommended partial factors for DA – 1 combination 1 and combination 2 are shown in Table 3 a). [3]

Table 3 a) Partial factors used for Design Approach 1 Combination 1

Material properties		
	Combination 1	Combination 2
$\gamma_{\varphi'}$	1.00	1.25
$\gamma_{c'}$	1.00	1.25
γ_{γ}	1.00	1.00

2. BACK CALCULATION OF OUR CASE STUDY

2.1 Back calculation using Probability Based Design Analysis

In Table 4 are presented the effective values of cohesion (c'), friction angle (φ') and unit weight of soil used to construct the embankment and of the soil of the first layer of the ground (which is also included in our model).

Table 4 Soil parameters

Material Properties	Embankment, soil type SM		First layer, Soil type CH
	Before failure	After failure	
φ' (°)	34	34	24.85
c' (kPa)	-	-	6.9
γ' kN/m^3	21.15	19.61	14.025

Probability based design analysis starts by estimating the standard deviations of the soil properties using the formula number (2): $\sigma_{\gamma'} = 0.476$ and $\sigma_{\varphi'} = 3.0$ for the embankment; $\sigma_{\gamma'} = 0.665$, $\sigma_{\varphi'} = 3.84$ and $\sigma_{c'} = 5.117$ for the first layer of the soil. We added one standard deviation to the average value of one parameter while all the other parameters are kept at their average value and we get the factor of safety (F^+) through the software analysis. We follow the same steps and subtracted the value of the standard deviation from the average value of the parameter to obtain the other value of the factor of safety (F^-). The results of this case study before the failure, under undrained conditions considering the underground water level 0.00 m are presented further in Table 5.

Table 5 Values of factors of safety and ΔF_n for the embankment before the failure

Soil parameter	Value	Factor of safety (F)		$\Delta F = \left\|F^+ - F^-\right\|$
Embankment (H=9 m)				
φ' (°)	$\varphi' + \sigma_{\varphi'} = 37$	F^+	1.448	0.079
	$\varphi' - \sigma_{\varphi'} = 31$	F^-	1.369	
γ' (kN/m³)	$\gamma' + \sigma_{\gamma'} = 21.626$	F^+	1.386	0.018
	$\gamma' - \sigma_{\gamma'} = 20.674$	F^-	1.404	
First layer of soil (H=3.50 m)				
φ' (°)	$\varphi' + \sigma_{\varphi'} = 28.69$	F^+	1.582	0.223
	$\varphi' - \sigma_{\varphi'} = 21.01$	F^-	1.297	
γ' (kN/m³)	$\gamma' + \sigma_{\gamma'} = 14.69$	F^+	1.563	0.367
	$\gamma' - \sigma_{\gamma'} = 13.36$	F^-	1.196	
c' (kPa)	$c' + \sigma_{c'} = 12.017$	F^+	1.420	0.049
	$c' - \sigma_{c'} = 1.783$	F^-	1.371	

Pursuant to the calculations using the formulas (5), (6) and (7) we derive $\sigma_F = 0.219$, $COV_F = 0.156$ (for F_{MLV}=1.406) and $\beta_{LN} = 2.123$. Via the Excel function "NORMSDIST", from the value of β_{LN} we get $R = 0.98$ and using the formula (1) $P_f = 0.02$.

Finally refereeing to the plot shown in Figure 3 and with the known values of P_f and COV_F is retrieved the factor of safety F = 1.38

The calculations and results of this case study after the failure under undrained conditions, considering the underground water level 0.00 m are presented in Table 6.

Table 6 Values of factors of safety and ΔF_n for the embankment after the failure

Soil parameter	Value	Factor of safety (F)		$\Delta F = \left\|F^+ - F^-\right\|$
Embankment (H=9 m)				
First layer of embankment from 0.00 to 1.00 (H =1 m)				
φ' (°)	$\varphi' + \sigma_{\varphi'} = 37$	F^+	1.451	0.044
	$\varphi' - \sigma_{\varphi'} = 31$	F^-	1.407	
γ' (kN/m³)	$\gamma' + \sigma_{\gamma'} = 21.626$	F^+	1.438	0.011
	$\gamma' - \sigma_{\gamma'} = 20.674$	F^-	1.427	

Second layer of embankment from 1.00 to 6.35 (H =5.35 m)				
φ' (°)	$\varphi'+\sigma_{\varphi'}=37$	F^+	1.460	0.074
	$\varphi'-\sigma_{\varphi'}=31$	F^-	1.386	
γ' (kN/m³)	$\gamma'+\sigma_{\gamma'}=19.980$	F^+	1.426	0.013
	$\gamma'-\sigma_{\gamma'}=19.234$	F^-	1.439	
Third layer of embankment from 6.35 to 9.00 (H =2.65 m)				
φ' (°)	$\varphi'+\sigma_{\varphi'}=37$	F^+	1.441	0.005
	$\varphi'-\sigma_{\varphi'}=31$	F^-	1.436	
γ' (kN/m³)	$\gamma'+\sigma_{\gamma'}=21.626$	F^+	1.438	0.009
	$\gamma'-\sigma_{\gamma'}=20.674$	F^-	1.447	
First layer of soil (H=3.50 m)				
φ' (°)	$\varphi'+\sigma_{\varphi'}=28.69$	F^+	1.555	0.211
	$\varphi'-\sigma_{\varphi'}=21.01$	F^-	1.344	
γ' (kN/m³)	$\gamma'+\sigma_{\gamma'}=14.69$	F^+	1.601	0.369
	$\gamma'-\sigma_{\gamma'}=13.36$	F^-	1.232	
c' (kPa)	$c'+\sigma_{c'}=12.017$	F^+	1.463	0.015
	$c'-\sigma_{c'}=1.783$	F^-	1.448	

Pursuant to the calculations using the formulas (5), (6) and (7) we derive $\sigma_F=0.217$, $COV_F=0.151$ (for F_{MLV}=1.437) and $\beta_{LN}=2.34$. Via the Excel function "NORMSDIST", from the value of β_{LN} we get $R=0.991$ using the formula (1) the result is $P_f=0.009$.
Finally refereeing to the plot shown in Figure 3 and with the known values of P_f and COV_F is retrieved the factor of safety F = 1.44

2.2 Back calculation using Eurocode 7

In this section of the paper we will solve this case according to the recommendations of Eurocode 7. The procedure consists of two steps. The first one is factoring all the soil parameters using the partial factors introduced in Table 3 a) and b). The second step concerns of inputting these values in the software and getting the factor of safety after running the analysis.

Table 7 Input parameters and results for the embankment according to DA – 1 combination 2 before failure

Input parameters before failure		
For embankment		
$\varphi_d^{'}=\tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right)$ = 28.35°	$\gamma_d=\frac{\gamma}{\gamma_\gamma}$ = 21.15 kN/m³	
First layer of soil		
$\varphi_d^{'}=\tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right)$ = 20.3 °	$\gamma_d=\frac{\gamma}{\gamma_\gamma}$ = 14.025 kN/m³	$c_d^{'}=\frac{c_k^{'}}{\gamma_{c'}}$ = 5.52 kPa

The safety factor taken by this analyses is F = 1.136

Table 8 Input parameters and results for the embankment according to DA – 1 combination 1 before failure.

Input parameters before failure		
For embankment		
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 34^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 21.15$ kN/m^3	
First layer of soil		
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 24.85^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 14.025$ kN/m^3	$c_d^{'} = \frac{c_k^{'}}{\gamma_{c'}} = 6.9$ kPa

The safety factor taken by this analyses is F = 1.406

Table 9 Input parameters and results for the embankment according to DA – 1 combination 2 after the failure.

Input parameters after failure		
For embankment		
First layer of embankment from 0.00 to 1.00 (H =1 m)		
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 28.35^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 21.15$ kN/m^3	
Second layer of embankment from 1.00 to 6.35 (H =5.35 m)		
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 28.35^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 19.61$ kN/m^3	
Third layer of embankment from 6.35 to 9.00 (H =2.65 m)		
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 28.35^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 21.15$ kN/m^3	
First layer of soil		
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 20.3^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 14.025$ kN/m^3	$c_d^{'} = \frac{c_k^{'}}{\gamma_{c'}} = 5.52$ kPa

The safety factor taken by this analyses is F = 1.164

Table 10 Input parameters and results for the embankment according to DA – 1 combination 1 after the failure.

Input parameters after failure	
For embankment	
First layer of embankment from 0.00 to 1.00 (H =1 m)	
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 34^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 21.15$ kN/m^3
Second layer of embankment from 1.00 to 6.35 (H =5.35 m)	
$\varphi_d^{'} = \tan^{-1}\left(\frac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 34^{o}$	$\gamma_d = \frac{\gamma}{\gamma_\gamma} = 19.61$ kN/m^3

<table>
<tr><td colspan="2">Third layer of embankment from 6.35 to 9.00 (H =2.65 m)</td></tr>
<tr><td>$\varphi_d^{'} = \tan^{-1}\left(\dfrac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 34^{o}$</td><td>$\gamma_d = \dfrac{\gamma}{\gamma_\gamma} = 21.15$ kN/m^3</td></tr>
</table>

<table>
<tr><td colspan="3">First layer of soil</td></tr>
<tr><td>$\varphi_d^{'} = \tan^{-1}\left(\dfrac{\tan\varphi_k^{'}}{\gamma_{\varphi'}}\right) = 24.85^{o}$</td><td>$\gamma_d = \dfrac{\gamma}{\gamma_\gamma} = 14.025$ kN/m^3</td><td>$c_d^{'} = \dfrac{c_k^{'}}{\gamma_{c'}} = 6.9$ kPa</td></tr>
</table>

The safety factor taken by this analyses is F = 1.437

CONCLUSIONS:

1. Our back calculation using Probability Based Design (PBDM) and EC-7 in this case study, gave us the following results for the safety factors:
   - Using PBDM: F = 1.380 which predicts a limit state of slope stability
   - Using EC-7, DA-1, Comb.2: F = 1.136 which predicts a failure state of the slope.
2. The failure surface in our case is an arc of a circle shaped (half-sine), whose action affects also the first layer of the natural ground.
3. The disagreement between the first safety factor calculated by Limit Equilibrum Methods (F=2.30) and our results is primarily due to the wrong determination of the characteristic strength values of the first layer of the ground. It is more convenient yo modelate the slope taking into consideration the strength parameters of the soil under undrained conditions.
4. Considering the fact that the first layer of the soil is sensitive clay we shall avoid the quick loading rate in these cases, in order to permit the excess pore water pressure to dissipate.
5. This case of study is a simple example that demonstrates the importance of the proper evaluation of soil parameters, proper way of loading the sensitive soils and the proper way of choosing the filling material for the embankment construction. [4]

REFERENCES

[1] Preloading to Mitigate Seismic Liquefaction Risk, Project No. 262161. Progress Report

[2] J. Michael Duncan and Stephen G. Wright (2005) Soil strength and slope stability. *John Wiley & Sons, Inc.,*USA.

[3] Eurocode 7: Geotechnical design (2004). *European Committee for Standardization.,* Brussels, Belgium.

[4] Braja M. Das. Principles of Geotechnical Engineering. *PWS Engineering, Boston.*