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Some general characteristics of Nakagami-m distribution

Hana Stefanovic[1], Ana Savic[1]

[1]College of Electrical Engineering and Computing Applied Science, Belgrade
Email: hanapopstefanovic@yahoo.com
Email: anas@viser.edu.rs

ABSTRACT

In this paper, study of general characteristics of the Nakagami-m probability density function is considered. For analytical and numerical evaluation of system performance, the Nakagami probability density functions are analyzed like particular solutions of corresponding differential equation, while the existence of singular solution is considered and analyzed under different conditions. It is also shown that the envelope of probability density function curves family is a straight line, for all values of varying parameters. This new expression can be used during mobile system designing.

INTRODUCTION

In mobile communications, a profound understanding and improvement of propagation channel models are very important for system design and related performance analysis. A great number of channel models have been proposed to describe the statistics of the amplitude and phase of multipath fading signals [1,2]. As the result of multipath reception, the mobile antenna receives a large number of reflected and scattered waves. Multipath propagation causes rapid fluctuations of the phase and amplitude of the received signal if the vehicle moves over a distance in the order of a wave length or more.

Depending on the nature of the propagation environment, there are different models describing the statistical behavior of the multipath fading envelope Rayleigh, Rician, Nakagami-m or Weibull model [3].

This paper discusses the case of Nakagami-m distribution of the received signal envelope, which models radio transmission in urban areas [1] where the random fluctuations of the instantaneous received signal power are very frequent and fast. Such Nakagami-m distribution can model different propagation conditions, providing more flexibility and higher accuracy in matching some experimental data than the commonly adopted distributions [4,5].

In the case of all distributions considered in the statistical theory of telecommunications, the pdfs of the received signal envelope are functions of several variables [6,7]. In the process of determination and analysis of integral characteristics, one of the variables is treated as a parameter, while the others are set to certain constant values of interest in practice. In this way, one obtains a family of signal envelope pdfs. Analysis of the position of the maximums for curves family can be performed analytically, using the first derivative of function, and also numerically. Then the same procedure is repeated for the case when the second variable is treated as a parameter, and the others are set to constant values. This process gives a new family of curves, while the maximum position is determined by the new envelope. In this paper the position of maximums and the analytical expression for the integral characteristic of the Nakagami-m distribution of received signal envelope are determined.

In order to improve performance of wireless communication systems, diversity techniques are used, which ensures that the receiver gets multiple copies of the radio signal transmitted by independent fading channels [8]. Increasing the number of independent paths for signal transmission significantly decreases the probability that in all independent channels simultaneously occurs significant fading of the signal. The main problem in designing of diversity systems is the choice of the optimal method for determining the final signal from the received replicas of transmitted signal [9]. For this purpose, the diversity systems use different techniques to combine signals: Selection Combining (SC), Maximum Ratio Combining (MRC), Equal Gain Combining (EGC), Switch and Stay Combining (SSC), Switch and Examine Combining (SEC). Determination of system performance in case of application of these techniques is widely discussed in the literature [10,11].

For most of these techniques, the choice of branch with the best signal-noise ratio is very important, because it is directly related to the value of signal envelope in this branch. Since in this paper the behavior of the maximum of signal envelope pdf is analyzed in detail, it could be applied to optimize the calculation of signal envelope at the combiner output.

The remainder of this paper is as follows. After this introduction, in Section II the procedure of determining integral characteristics of distribution is considered. Analytical and numerical resulst showing that the envelope of family of Nakagami-m pdf curves exists, are presented in Section III. Conclusion is given in Section IV.

THE PROCEDURE OF DETERMINING INTEGRAL CHARACTERISTICS OF DISTRIBUTION

The previous analysis of probabilistic models for the description of the received signal envelope, depending on the individual relevant parameters of the transmission fading channel, resulted in idea to focus the subject of research at the request of the integral description of phenomena that affect the radio signal transmission. We seek to determine analytical expressions for description of boundary receiving conditions that could be useful in the design of radio receiving systems.

For the considered Nakagami-m model, for a fixed value of the fading depth parameter m, the pdf of signal envelope, depending on the received signal level, is analyzed for different values of average signal power [12]. Then the average signal power was treated as parameter, while the values of the received signal level are varied. In such a way, two series of families of curves are obtained and for each of them equation of the envelope of curves maximums is constructed. In both cases these envelopes are straight lines, in the logarithmic scale, whose direction coefficients and values on ordinate-axis are determined analytically and numerically. Then the same procedure is repeated for different values of the fading depth parameter. It is found that obtained series of envelopes have the same direction coefficient.

The obtained numerical, graphical and analytical results indicate the existence of differential equation whose particular solution is just the pdf of the observed distribution. Envelope of maximums of family of pdf curves represents the singular solution of differential equation [13] describing the dynamics of the process of signal transmission in fading channel.

INTEGRAL CHARACTERISTICS OF NAKAGAMI-M DISTRIBUTION

Nakagami-m distribution is suitable for describing statistics of mobile radio transmission in complex medium such as the urban environment [14]. In practice it has proved very useful because of easy manipulation and wide range of applicability of various approximations. Since the Nakagami-m random process is defined as envelope of the sum of 2m independent Gauss random processes, the Nakagami-m distribution is described by pdf [14,15]:

$$p_z(z,\Omega)=\frac{2}{\Gamma(m)}\left(\frac{m}{\Omega}\right)^m z^{2m-1}\exp\left(-\frac{m}{\Omega}z^2\right), z>0, m\geq\frac{1}{2} \tag{1}$$

where z is the received signal level, $\Gamma(\cdot)$ is gamma function, m is the parameter of fading depth (fading figure), defined as:

$$m=\frac{E^2[z]}{Var[z^2]} \tag{2}$$

while Ω is average signal power:

$$\Omega=E[z^2]. \tag{3}$$

In the case when the parameter of fading depth is m=1 Nakagami-*m* distribution reduces to the familiar Rayleigh distribution [1], while the case m=0.5 corresponds to the unilateral Gauss distribution. Case $m\rightarrow\infty$ describes the channel without fading. With certain restrictions Nakagami-*m* distribution can approximate Rice distribution [14]. Nakagami-*m* channel model in the analytical sense is simpler than Rice’s, in which appears Bessel function, so that using the above approximation calculation of statistical characteristics significantly simplifies. We choose to analyze Nakagami-*m* channel model for reasons of generality, and because the other models can be described by the Nakagami-*m* distribution by appropriate choice of relevant parameters.

Graphic presentation of dependence of Nakagami-*m* pdfs versus received signal level, for a fixed value of the fading depth parameter and different values of average signal power, in logarithmic scale, is shown in Fig. 1.

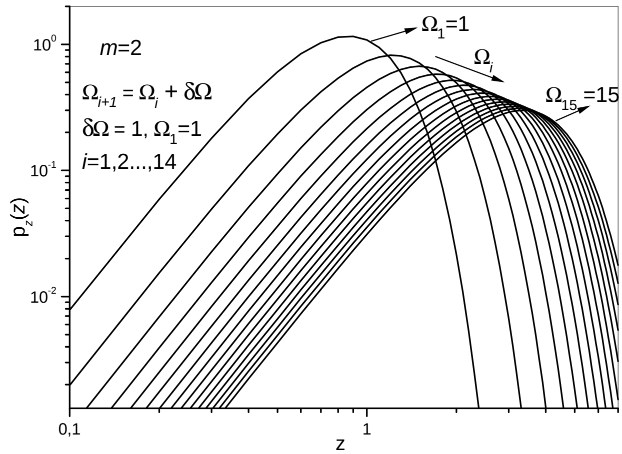

Figure 1. The Nakagami-*m* pdf versus *z*, in logarithmic scale, for case *m*=2, with Ω taking values from 1 to 15

Analysis of this dependence shows that the increase of average signal power the pdf maximums result in smaller values reached for larger values of received signal level, as shown in Fig. 1. It can also be concluded that all the maximums lie on a straight line, and that the envelope of maximums is also a straight line, in logarithmic scale. In order to determine the values of received signal level when the maximums are reached, as well as the values of maximums and direction coefficient of envelope, the first derivative of (1) relative to received signal level is determined. Equalization with zero obtains:

$$z_{p\max} = \sqrt{\frac{\Omega(-1+2m)}{2m}} \tag{4}$$

Substituting (4) into (1) yields:

$$p_z^{z\max} = \frac{2^{1+\frac{1-2m}{2}}}{\Gamma(m)} \cdot \left(\frac{m}{\Omega}\right)^m \left(\frac{\Omega(2m-1)}{m}\right)^{\frac{2m-1}{2}} \cdot \exp\left(\frac{1-2m}{2}\right) \tag{5}$$

From (5) we get:

$$\log\left(\max p_z(\Omega)\right) = k_1 \log \Omega + n_1, \tag{6}$$

where the direction coefficient and the value on the ordinate axis are:

$$k_1 = -0.5,\ n_1 = -m + 0.5 + \log\frac{2m^m}{\Gamma(m)} + (m-0.5)\log\frac{2m-1}{2m}. \tag{7}$$

Direction coefficient of envelope has the value -0.5, regardless of the value of the fading depth parameter, while the value on the ordinate axis depends on this parameter. Envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the pdf is its particular solution:

$$p_z'(z)+p_z(z)\left(\frac{2mz}{\Omega}-\frac{2m-1}{z}\right)=0\ . \tag{8}$$

Graphic presentation of dependence of Nakagami-*m* pdfs versus average signal power, for a fixed value of the fading depth parameter and different values of received signal level, in logarithmic scale, is shown in Fig. 2.

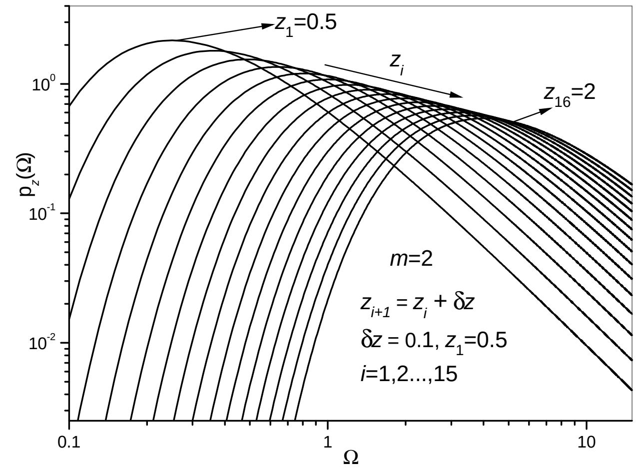

Figure 2. The Nakagami-*m* pdf versus Ω, in logarithmic scale, for case *m*=2,with *z* taking values from 0.5 to 2

Analysis of this dependence shows that with increase of received signal level the pdf maximums achieve smaller values for larger values of average signal power, as shown in Fig. 2. It can also be concluded that all the maximums lie on a straight line, and that the envelope of maximums is also a straight line, in logarithmic scale. In order to determine the values of average signal power when the maximums are reached, and the values of maximums and direction coefficient of envelope, the first derivative of (1) relative to average signal power is determined. After equalization with zero one obtains:

$$\Omega_{p\max} = z^2 \tag{9}$$

Substituting (9) into (1) yields:

$$p_z^{\Omega\max} = \frac{2m^m}{z \cdot \Gamma(m)} \cdot \exp(-m) \tag{10}$$

From (10) we get:

$$\log\left(\max p_z(z)\right) = k_2 \log z + n_2, \tag{11}$$

where the direction coefficient and the value on the ordinate axis are:

$$k_2 = -1,\ n_2 = \log \frac{2m^m}{\Gamma(m)} - m\ . \tag{12}$$

Direction coefficient of envelope has the value -1, regardless of the value of the fading depth parameter, while the value on the ordinate axis depends on this parameter. Envelope determines a certain singular solution of differential equation which can describe the dynamics of this process, while the pdf is its particular solution:

$$p_z'(\Omega)+p_z(\Omega)\left(1-\frac{z^2}{\Omega}\right)\cdot\frac{m}{\Omega}=0\,. \tag{13}$$

Previously described analysis was repeated for different values of the fading depth parameter, to discuss how this parameter affects on the shape of the envelope of pdf curves maximums. In the case *m*=0.7, obtained results are shown in Fig. 3 and Fig. 4., respectively.

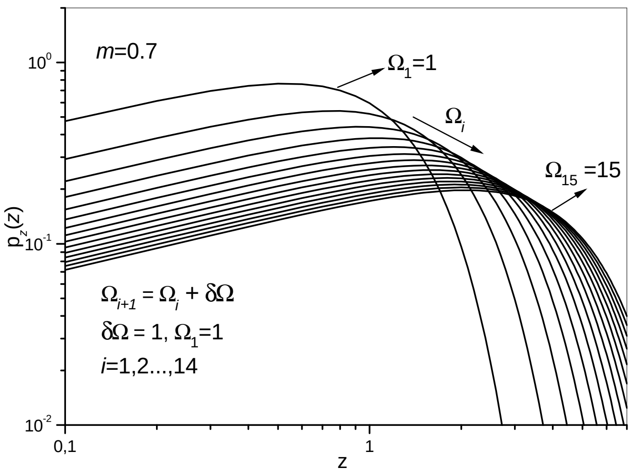

Figure 3. The Nakagami-*m* pdf versus *z*, in logarithmic scale, for case *m*=0.7,with Ω taking values from 0.5 to 2

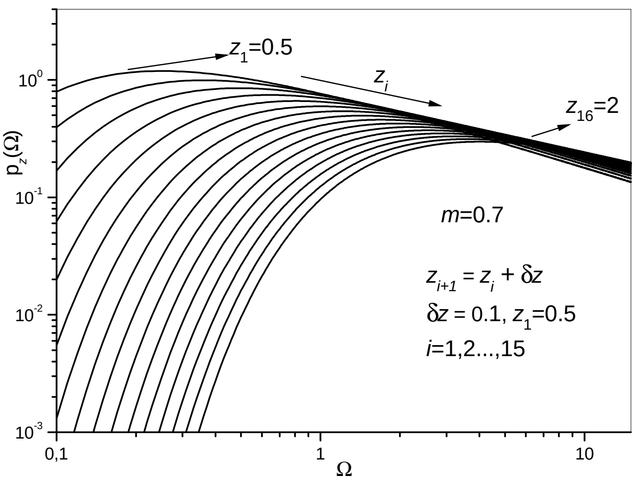

Figure 4. The Nakagami-*m* pdf versus Ω, in logarithmic scale, for case *m*=2,with *z* taking values from

0.5 to 2

It can be concluded that the envelopes of pdf maximums in both cases are straight lines in logarithmic scale. Direction coefficients of obtained envelopes are the same as in the case m=2, only the value on the ordinate axis changes. In analysis of dependence of Nakagami-m pdfs versus average signal power, as in analysis of dependence of Nakagami-m pdfs versus received signal level, the values on the ordinate axis are smaller.

Envelopes of pdf maximums for different values of fading depth parameter when the received signal level is taken as a parameter, as well as in the case when the average signal power is taken as a parameter, are graphically presented in Fig. 5 and Fig. 6., in logarithmic scale.

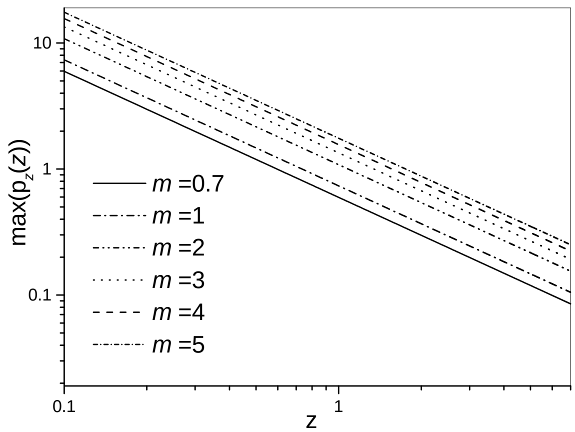

Figure 5: The maximums of Nakagami-m pdfs versus z, in logarithmic scale,for different values of fading depth parameter

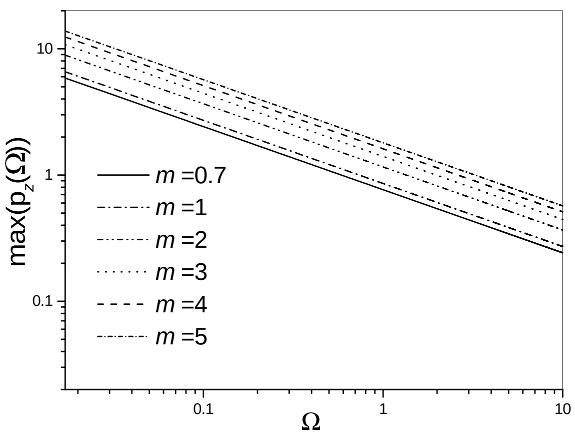

Figure 6. The maximums of Nakagami-*m* pdfs versus Ω, in logarithmic scale,for different values of fading depth parameter

Analyzing the dependence of envelope of Nakagami-*m* pdfs versus received signal level, for different values of fading depth parameter, one concludes that all envelopes have the same direction's coefficient, regardless of the value of fading depth parameter, while their values on the ordinate axis increase with increase of fading depth parameter, as shown in Fig. 5.

When analyzing the dependence of envelope of Nakagami-*m* pdfs versus average signal power, for different values of fading depth parameter, it can be concluded that all envelopes have the same direction's coefficient, regardless of the value of fading depth parameter, while their values on the ordinate axis increase with increase of fading depth parameter, as shown in Fig. 6.

A three-dimensional graphs showing the dependency of Nakagami pdf versus different parameters are presented on Fig 7. and Fig. 8.

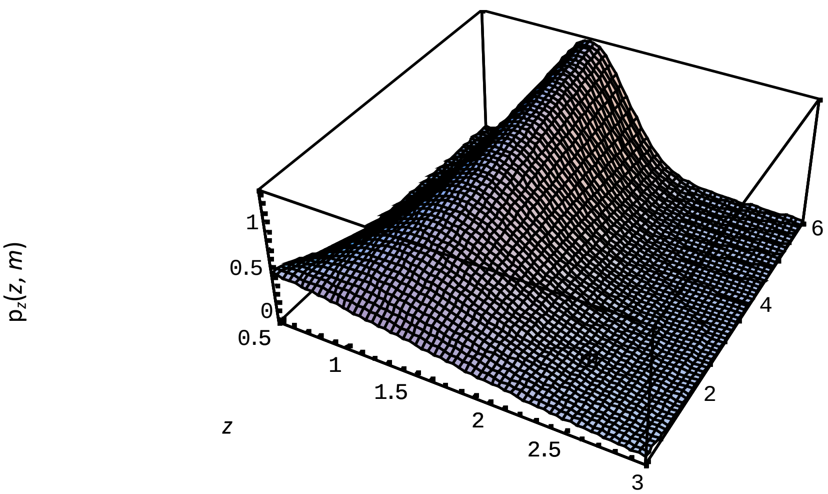

Fig. 7. A three-dimensional graph showing the dependency of Nakagami-*m* pdf versus *z* and *m* for Ω=2

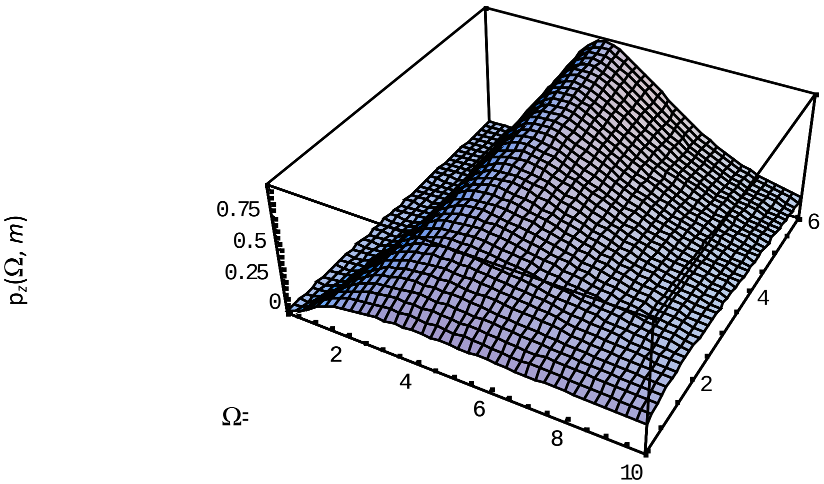

Fig. 8. A three-dimensional graph showing the dependency of Nakagami-*m* pdf versus Ω and *m* for *z* =2

CONCLUSION

In order to determine the first-order and the second-order statistics of multipath fading channel models, it is necessary to know the characteristics of pdf and its envelope. This paper presents the results of testing properties of envelope of pdf curves family, which until now were not treated in the literature.

The analytical expression for the envelope of pdf curves family maximums is determined depending on the relevant parameters. The obtained results show that the position of the maximums of these functions is uniquely determined by envelope's equation, regardless of the values of the other parameters. In such a way, boundary conditions for radio transmission, with given propagation conditions, can be defined.

For the considered multipath fading channel model, differential equation describing the complete dynamics of the signal transmission process is determined, whereby envelope of pdf curves family represents its singular solution.

Derived analytical expressions could be used during the process of radio links modeling and evaluation of system performance, since almost all indicators of system performance are related to pdf.

REFERENCES

[1] J. Proakis, Digital Communications, 3rd ed., McGraw-Hill, 1999.

[2] D. Drajic, Introduction to statistical theory of telecommunications, Academic Science, Belgrade, 2003.

[3] W. Y. C. Lee, Mobile Cellular Communications, McGraw-Hill, New York, 1989.

[4] H. Suzuki, "A statistical model for urban multipath propagation", *IEEE Trans. Commun.*, vol. COM-25, July 1977.

[5] T. Aulin, "Characteristics of a digital mobile radio channel", *IEEE Trans. Veh. Technol.*, vol. VT-30, May 1981.

[6] R. H. Clarke, "A Statistical Theory of Mobile Radio Reception", Bell Systems Technical Journal, vol. 47, pp. 957-1000, 1968.
[7] R. Janaswamy, Encyclopedia of RF and Microwave Engineering, University of Massachusetts, Amherst, USA.
[8] G. K. Karagiannidis, "Moments-Based Approach to Performance Analysis of Equal Gain Diversity in Nakagami-*m* Fading", IEEE Transactions on Communications, vol. 52, No. 5, pp. 685-690, May 2004.
[9] M. K. Simon, M. S. Alouni, Digital Communications over Fading Channels: A Unified approach to performance analysis, John Wiley, New York, 2000.
[10] G. Femenias, "BER Performance of Linear STBC From Orthogonal Designs Over MIMO Correlated Nakagami-m Fading Channels", IEEE Transactions on Vehicular Technology, vol. 53, No. 2, pp. 307-317, March 2004.
[11] H. Shin, J. H. Lee, "On the Error Probability of Binary and M-ary Signals in Nakagami-*m* Fading Channels", IEEE Transactions on Communications, vol. 52, No. 4, pp. 536-539, April 2004.
[12] H. Popovic, D. Stefanovic, A. Mitic, I. Stefanovic, "Some Statistical Characteristics of Nakagami-*m* distribution", TELSIKS 2007, Proceedings of Papers, vol. 2, pp. 509-512, Nis, Serbia, 2007.
[13] A. D. Polyanin, V. F. Zaitsev, A. Moussiaux, Handbook of First Order Partial Differential Equations, Taylor & Francis, London, 2002.
[14] M. Nakagami, The m-distribution, a General Formula of Intensity Distribution of Rapid Fading in Statistical Methods in Radio Wave Propagation, W. G. Hoffman, Ed., Pergamon, Oxford, England, 1960.
[15] N. Youssef, T. Munakata, M. Takeda, "Fade Statistics in Nakagami Fading Environments", Proc. of 4th IEEE Int. Symp. "Spread Spectrum Techniques & Applications", pp. 1244-47, Mainz, Germany, 1996.